"The mathematics of tube inflation".

Abstract :

Inflation of a membrane tube is a classical problem but has received renewed interest in recent years. It is well-known that when one end of the tube is closed and is allowed to extend freely during inflation, the pressure vs volume curve corresponding to uniform inflation has an up down-up behaviour. This explains the phenomenon that when a tubular party balloon is inflated, it would quickly evolve into a "multi-phase" state: the coexistence of a thick section and a thin section. Another less understood loading scenario that may be more relevant in hi-tech applications is when the tube is first stretched axially to a specified length and then inflated with both ends fixed. We report our recent studies in which we treat localised bulging as a bifurcation problem and are able to give the entire inflation process a complete analytical description for both loading scenarios. We wish to promote this problem as a textbook example to illustrate a variety of important concepts and techniques in mathematics and mechanics such as strain localisation, snap-through, phase transition, and self-consistent asymptotic derivation of 1D models, etc. We also show how understanding the tube inflation problem has guided our studies of other more challenging elastic localisation problems such as the axisymmetric (thickness) necking of a radially stretched circular membrane.