

“A free boundary approach of spectral optimization problems”

Abstract :

Why all the drums are round! This assertion is mathematically rephrased as follows: among all membranes with prescribed area, the one producing the lowest fundamental frequency has a circular shape. But what about the shapes of membranes with higher extremal frequencies? The general question in which we are interested concerns the relationship between the geometry of the domain and the spectrum of a differential operator defined on this domain. In this talk I will focus on the Neumann eigenvalues of the Laplace operator on domains of Euclidean space and on spheres. Together with a discussion about existence/non existence of optimal geometries and possible relaxation on densities, I will give some numerical approximations of the best geometries and densities. A particular attention will be given to the lowest two non trivial eigenvalues for which a full answer is given in any dimension of the Euclidean space. A surprising phenomenon occurs on spheres: while a complete answer can be given for the second eigenvalue, for the first one an unexplained phenomenon occurs. The results presented in this talk have been obtained in series of collaborations with R. Laugesen, A. Henrot, E. Martinet, M. Nahon and E. Oudet.