

INDAM-COFUND-2012

Project Title: *A Duality-based Approach To Deductions And Reasoning with Vague Information*

Fellow: Diego Valota

In the original proposal the 24-months period of the project was divided in four phases, one for each objective:

O1 - using duality theorems and other constructions to constructively study unification and interpolation problems for specific many-valued logics;

O2 - developing algorithms to build unifiers and interpolants to bring the expressive power of these logics to real-world applications;

O3 - using dualities to characterize residuated frames for the studied logics and generalize the developed methods to more general structures;

O4 - using residuated frames as a bridge to apply the developed reasoning techniques to the probabilistic setting.

Before the beginning of this project, the research fellow has started an investigation on formal contexts related to many-valued logics in collaboration with P. Codara from University of Milan (the host institution designed for the returning phase). In order to pursue this research direction the project timeline has changed accordingly, we started investigating O3. Indeed, formal contexts are relational semantics that can be seen as special cases of (finite) frames. Preliminary results on this research have been presented during a seminar [T2] at the University of Milan, and in two contributed talks [T4, T5]. Finally, the research has been published [CV17] in the post-proceedings of the TbiLLC conference.

This research direction appears particularly fruitful. Indeed, the bridge role of these relational structures (O4) is confirmed by looking at them from the point of view of duality theory. Considering a variety of algebras corresponding to a many-valued logic, it turns out that a formal context is an instance of a product fibred over the dual of the free 1-generated algebra in the variety. Amalgams of algebras (the algebraic equivalent of logical interpolants) in dual spaces are exactly fibred products. Then, the algorithmic characterization of amalgamation properties (O2) will be a useful tool also for the study of the relational structures planned in O3.

In this sense, the fellow has obtained a combinatorial procedure to build uniform interpolants for Nilpotent Minimum logic. Preliminary results on this construction have been presented as a contributed talk [T6] at the ESTYLF conference. A paper [V17] with the full construction has been published in the proceedings of the ISMVL2017 conference.

Finally, we mention that thanks to the interests around these topics the fellow has been invited at the workshops [W1] and [W2] held at the Lorentz Center in Leiden.

In parallel with the above sketched investigations, the fellow has started to approach O1, introducing part of his research plan to the local logic community during a seminar [T1] at the University of Barcelona (May 15, 2015). The fellow together with the logic research group at the IIIA (the outgoing host institution), has started to study the relation between uniform interpolation problem and propositional quantified formulas in many-valued logics. This latter approach represents an integration between the fellow's project and the logic research carried out at the IIIA. In January 2017 the fellow has visited IIIA and University of Valencia, to work with Lluís Godo and Félix Bou respectively [V1]. Research on these topics is still ongoing.

As mentioned in the project proposal, the study of combinatorial representations and dualities for many-valued logics is a useful tool to approach O1 and O2. In this sense, the fellow in collaboration with S. Aguzzoli (University of Milan) and S. Bova (Technische Universität Wien) has studied the many-valued logic of weak negations (WNM). Preliminary results of this research have been presented [T3] at the conference "Topology, Algebra, and Categories in Logic" (Ischia, June 25, 2015). The final work has been published as [ABV17].

During the return phase, the fellow together with the research group at the returning host institution, has started to face O4 by studying dualities for many-valued logics in order to define probability measures over many-valued logical formulas. This technique has been first applied to Gödel-Delta logic, and the results have been

presented to the international conference ECSQARU2017 in [T9], and published in [ABGV17].

As complementary activities, the fellow has been part of the organization of a theoretically-oriented special session at the computer science conferences: FUZZ-IEEE 2015 [C1] and WCCI2016 [C2]. The fellow was a member of the program committee of both editions.

During the INDdAM-COFUND fellowships, a European RISE project called SYSMICS [P1] involving IIIA and University of Milan has been approved. As both IIIA and University of Barcelona are SYSMICS members, it has been decided to hold the kick-off project meeting [O1] in Barcelona. The fellow was an active member of the organizing committee. Another conference organized by the fellow was [O2], held in Milan in honour of Daniele Mundici's 70th birthday. Finally, the fellow has also attended the workshop [A1].

Research Activities:

Published Papers:

[ABV17] Free weak nilpotent minimum algebras. S. Aguzzoli, S. Bova, and D. Valota.

Soft Computing. Volume 21(1), pages 79–95. 2017. ISSN 1433-7479.

[V17] Computing Uniform Interpolants in Nilpotent Minimum Logic. D. Valota

47th IEEE International Symp. on Multiple-Valued Logic (ISMVL), ISSN 2378-2226, pages 242-247, 2017.

[CV17] On Gödel Algebras of Concepts. P. Codara, and D. Valota.

Logic, Language, and Computation, Lecture Notes in Computer Science, Volume 10148, pages 251-262, 2017.

[ABGV17] Probability measures in Gödel-Delta logic. S. Aguzzoli, M. Bianchi, B. Gerla, and D. Valota.

ECSQARU 2017. Lecture Notes in Computer Science, vol 10369. pages 353-363, 2017

Talks:

[T9] Probability measures in Gödel-Delta logic.

ECSQARU2017, Lugano, July 13, 2017.

[T8] Computing Free Spectra of Many-Valued Logics.

93rd Workshop on General Algebra. Bern, February 11, 2017.

[T7] Finite Forests: Logical Meaning, Counting Problems and Generalizations.

Seminar for the course of Graph Theory, Prof. O.D'Antona. Milan, January 12, 2017.

[T6] Uniform Interpolants in Nilpotent Minimum Logic.

XVIII Congreso Español sobre Tecnologías y Lógica Fuzzy. ESTYLF 2016. Donostia-San Sebastian, May 27, 2016.

[T5] Towards a Formal Concept Semantics for Gödel Logic.

11th International Tbilisi Symposium on Language, Logic and Computation. Tbilisi, September 25, 2015.

[T4] Formal Concepts of Gödel Logic Formulas.

Trends in Logic XV. Delft, July 2, 2015.

[T3] Representation of Free Finitely Generated Weak Nilpotent Minimum Algebras.

TACL 2015. Ischia, June 25, 2015.

[T2] Contexts of Finite Gödel Algebras: First Steps on Characterization and Computation.

Logic Seminar. University of Milan, June 12, 2015.

[T1] Computing Amalgams and Interpolants in Finite Gödel Algebras via Duality.

Non-Classical Logics Seminar. University of Barcelona, May 15, 2015.

Invited Participant:

[W2] Workshop on Logics, Decisions, and Interactions. October 24-28, 2016, Leiden, Netherlands

[W1] Unified Correspondence Workshop. February 15-19, 2016, Leiden, Netherlands

Visiting:

[V1] January 24th-27th 2017, Lluís Godó (IIIA), January 22th-24th 2017, Felix Bou (University of Valencia),

Complementary Activities:

Organising Committee:

[O2] MAGICA16, September 26-27, 2016, Milan, Italy

[O1] Syntax Meets Semantics 2016. September 5-9, 2016, Barcelona, Spain

Program Committee, Special Session Organiser:

[C2] WCCI 2016, July 25-29, 2016, Vancouver, Canada

[C1] FUZZ-IEEE 2015, August 2-5, 2015, Istanbul, Turkey

Attendee:

[A1] Prague Seminar: The Future of Mathematical Fuzzy Logic. June 16-18, 2016, Prague, Czech Republic.

Projects:

[P2] “Modalities through Dualities and Generalised Relations – MoDulaR”. INdAM Young researchers funds (proponent, awaiting evaluations).

[P1] SYSMICS: Syntax meets Semantics – Methods, Interactions, and Connections in Substructural logics. H2020-MSCA-RISE-2015, grant agreement No 689176 (member).

Career Development and Collaborations:

The fellow is now a postdoctoral researcher at the Department of Computer Science of the University of Milan (the return phase host institution), where he had the opportunity to be teaching assistant for the undergraduate course in Logic for computer scientists. Moreover, he serves also as Lecturer on Contract at the Department of Mathematics, for the undergraduate course in Coding for the mathematics degree.

The collaboration with the IIIA is still ongoing. Indeed, the fellow together with the IIIA logic researcher has recently submitted a paper on the relations between formal concept analysis and many-valued logics. Moreover, the study on relations between uniform interpolation problem and propositional quantified formulas in many-valued logics started at the IIIA with L. Godo and F. Bou (now at the University of Valencia) is still in progress.

Thanks to the knowledge earned during this investigation, the fellow has also started its own research program facing spectra problems for many-valued logics via dualities. In this sense, during [W2] the fellow has started a collaboration with S. Bonzio (University of Ancona) to study analogous problems for the weak Kleene logic. M. Pra Baldi (University of Padova) has now joined this project.

The success of the special sessions at [C1] and [C2], has prompted the organization of other ones for the 2017 and 2018 editions of FUZZ-IEEE and WCCI conferences. The fellow is the chair of the special session at WCCI2018.

By participating to several conferences and workshops during the INdAM-COFUND fellowship, the fellow had the opportunity to broaden his knowledge of nonclassical logics, to look at different research directions, and to get inspired by prominent researchers of the field. Thanks to this motivating situations, the fellow has been stimulated to write a project proposal on a novel approach to introduce modalities for nonclassical logics via dualities [P2].

Finally, the fellow has joined the SYSMICS project [P1], and then he is expected to visit the logic group of the University of Campinas in Brazil for a month during 2018.