Final report for INdAM president on the project INdAM-Cofund 2012

Andrea Santi

21st February 2017

Employment

Marie-Curie Fellow of INdAM 1st February 2015 - 31st January 2017

The University of Bologna, Italy (from 1st August 2016)
The University of Edinburgh, UK (until 31st July 2016)
Investigator on the project "The geometry of supermanifolds"

Research Works

(1) *Sasakian manifolds and M-theory* [with J. Figueroa-O'Farrill, University of Edinburgh], Classical Quantum Gravity **33** (2016), 095004 (18pp).

This paper extends the link between Sasaki-Einstein manifolds and Killing spinors to a class of η-Sasaki-Einstein manifolds, both in Riemannian and Lorentzian settings, characterising them in terms of generalised Killing spinors. A definition of supersymmetric M-theory background is then proposed and a new class of such backgrounds determined.

(2) *Homogeneous irreducible supermanifolds and graded Lie superalgebras* [with D. Alekseevsky, University of Moscow], Int. Math. Res. Not. (2016), doi: 10.1093/imrn/rnw262 (35pp).

In this paper we initiate the study of the geometry of supermanifolds. We consider the case of isotropy irreducible homogeneous supermanifolds and obtain the full classification of associated $\mathbb{Z}$-graded Lie superalgebras admitting "non-linear" symmetries.

(3) *Homogeneous models for Levi degenerate CR manifolds*, preprint arXiv:1511.08902, 2016 (40pp). Submitted for publication.

This work deals with submanifolds of complex manifolds with degenerate Levi forms. We propose a definition of homogeneous model in the spirit of Tanaka for k-nondegenerate CR manifolds, $k \geq 2$, and investigate their properties. The general theory is applied to the case of 7-dimensional CR manifolds of hypersurface type, where seven different 2-nondegenerate classes of homogeneous models and one 3-nondegenerate class are obtained.

(4) *Spencer cohomology and* 11*-dimensional supergravity* [with J. Figueroa-O'Farrill], Comm. Math. Phys. **349** (2017), 627–660.

We recover the classification of maximally supersymmetric solutions of 11-dimensional supergravity using the framework of Tanaka structures on supermanifolds. In particular we classify the filtered deformations of the Poincaré superalgebra $\mathfrak{p}$ and show that there are only three other Lie superalgebras, the symmetry superalgebras of the non-flat maximally supersymmetric backgrounds. We identify the gravitino variation with (a component of) a Spencer cocycle of $\mathfrak{p}$.

(5) *Eleven-dimensional supergravity from filtered subdeformations of the Poincaré superalgebra* [with J. Figueroa-O'Farrill], J. Phys A: Math. Theor. **49** (2016), 295204 (7pp).

The results of (4) are summarized, highlighting what might be considered to be a Lie-algebraic derivation of 11-dimensional supergravity. In view of works (4)-(5) and their applications (cf. (6) and (7)), I organized an internal working seminar at the University of Edinburgh (see later).

(6) *Killing superalgebras for Lorentzian four-manifolds* [with P. de Medeiros, University of Cardiff, and J. Figueroa-O'Farrill], J. High Energy Phys. **6** (2016), 2016:106 (50 pp).

This paper studies Killing superalgebras of 4-dimensional Lorentzian manifolds using the cohomological approach developed in (4)-(5). The generalized Killing spinor equations and the Lorentzian manifolds we determined are relevant to the construction of rigid supersymmetric theories.

(7) *On the algebraic structures of Killing superalgebras* [with J. Figueroa-O'Farrill], preprint arXiv:1608.05915, 2016 (28pp). Submitted for publication.

The algebraic structure of the Killing superalgebra of an 11-dimensional supergravity background is further investigated and the classification problem for backgrounds is re-interpreted as classification problem of certain admissible filtered subdeformations of the Poincaré superalgebra. In particular we show that the Einstein and Maxwell equations are automatically satisfied for backgrounds preserving more than half the supersymmetry.

Works in preparation

(8) *Classification of linearly compact simple Kantor triple systems* [with N. Cantarini, A. Ricciardo, University of Bologna], under completion.

We fully classify finite and infinite dimensional Kantor triple systems over algebrically closed fields – in particular, we classify and explicitly describe all the Kantor triple systems of exceptional type. In view of this work and its future applications and generalizations, I organized an internal working seminar at the University of Bologna (see later).

(9) *Six-dimensional Killing spinors from Spencer cohomology* [with P. de Medeiros, J. Figueroa-O'Farrill], in progress.

The structure of the Killing superalgebras of 6-dimensional Lorentzian manifolds is investigated.

Teaching experience

Working Seminar, University of Edinburgh:

Responsible of the working seminar (16 hrs) *The geometry of supergravity* [with J. Figueroa-O'Farrill], A.Y. 2015-2016, for a personal total of 12 hrs of lectures. (Tanaka structures, the gap phenomenon in parabolic geometries, supergravity; see http://empg.maths.ed.ac.uk/Activities/SUGRA/index.html).

Working Seminar, University of Bologna:

Responsible of the working seminar (6hrs) *Supergravity, $\mathbb{Z}$-graded Lie superalgebras and generalized Kantor triple systems*, A.Y. 2016-2017, for a personal total of 6 hrs of lectures. (Supergravity, Kantor triple systems, $N = 5$ three-algebras; see https://www.dm.unibo.it/seminari/cycles/23).

Supervision experience

I proposed the summer project *The algebra of the standard model* to 3rd year student P. Hannay of the Department of Mathematics and Physics of the University of Edinburgh. The project has been funded by an EPSRC Scholarship and carried out during June-July 2016.

Invited talks at conferences and seminars

– *On the algebraic structure of Killing superalgebras*
Geometry and Topology Seminar, University of Hradec Kralove (CZ), 15th December 2016

– *Killing superalgebras and high supersymmetry*
FSG Seminar, University of Surrey (UK), 29th November 2016

– *Killing superalgebras and high supersymmetry*
EMPG Seminar, University of Edinburgh (UK), 23rd November 2016

– *Killing superalgebras and filtered deformations*
Conference "Superstring solutions, supersymmetry and geometry" (Spain), 1st-7th May 2016

– *On Killing superalgebras and filtered deformations*
London Triangle Seminar, King's College (UK), 30th March 2016

– *On Spencer cohomology and maximal supersymmetry*
Seminars of the Edinburgh Mathematical Physics Group, Edinburgh (UK), 25th November 2015.

– *On Spencer cohomology and maximal supersymmetry*
45th Meeting of the North British Mathematical Physics Seminars, University of Durham (UK), 21st November 2015.

– *On extended Poincaré structures and maximal supersymmetry*
Workshop "A Lie day in Bologna", University of Bologna (Italy), 1st October 2015.

Invited research stays

– University of Hradec Kralove (Czech Republic) 12th - 16th December 2016

– University of Edinburgh (UK) 21st - 25th November 2016

Other professional activity

– Referee for J. Geom. Phys., Int. J. Geom. Methods Mod. Phys and SIGMA.

– Reviewer for the AMS publication "Mathematical Reviews".

Firenze, 21st February, 2017

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