Annual Activity Report Year 1
Yukihide Nakada
Advisor: Prof. Bruno Chiarellotto

1 Coursework

All courses have taken place at the University of Padova, unless otherwise specified.

Winter Semester 2018

1. Lie Groups and Symmetry, Prof. Francesco Fassò: passed
2. Representation Growth and Zeta Functions, Prof. Benjamin Klopsch: passed

Summer Semester 2019

1. The Fundamental Group, Prof. Bruno Chiarellotto and Prof. Adrian Iovita: passed
2. Foundations of Data Analysis, Prof. Massimo Fornasier: passed
3. Elliptic Curves and Modularity, Prof. Bao Viet Le Hung: passed

Winter 2019

1. No courses

Summer 2020

1. The Arithmetic of Elliptic Curves, Prof. Bruno Chiarelloto and Prof. Adrian Iovita: In progress

2 Seminars, Workshops, and Conferences

1. Winter Semester 2018 - Summer Semester 2019: Seminar “From Berkovich to Moduli”. Padova, Italy.
2. November 18 - 24, 2018: Oberwolfach Seminar: Syntomic Cohomology and p-adic Hodge Theory. Oberwolfach, Germany.
3. February 7 - 13, 2019: Minicourse on motivic homotopy theory. Verona, Italy.
4. May 27 - June 14, 2019: INdAM Summer School on the Serre conjectures and the p-adic Langlands program. Padova, Italy.
5. July 18 - 25: Seminar in Logarithmic Algebraic Geometry. Münich, Germany. Talk: “Monoids, log structures, and log schemes”.
6. September 18 - 20, 2019: Over and around sites in characteristic p, in honor of Bernard le Stum. Padova, Italy.
7. January 21 - 31, 2020: Moduli Day. Münich, Germany.
8. March 16 - 20, 2020: Conference on Arithmetic Algebraic Geometry. Darmstadt, Germany.
9. April 6 - 10, 2020: Périodes, motifs et équations différentielles, entre arithmétique et géométrie. Paris, France.

3 Papers

None completed

4 Visiting Periods

1. January 14 - 18, January 23 - February 2. Collaboration with advisor Prof. Chiarellotto at the Institute Des Hautes Études Scientifiques. Orsay, Paris.

2. January 21 - 31: Collaboration with Prof. Christian Liedtke. Münich, Germany

5 Description of Research Done

This year, I primarily focused on understanding the Hyodo-Steenbrink double complex as defined by Mokrane in [Mok93] and studying the applications of duality theory as in [Har66] and [ElZ83] to complexes induced by the Hyodo-Steenbrink complex. More specifically, Chiarellotto proved in [Chi99] that the kernel of the monodromy endomorphism on the Hyodo-Steenbrink double complex on a semi-stable log scheme, which induces the usual monodromy on its log-crystalline/Hyodo-Kato cohomology, computes the *rigid* cohomology of the variety. Pointing towards an application to duality, Mokrane showed that on each page of the canonical spectral sequence which computes the cohomology of the total complex of the Hyodo-Steenbrink double complex we have a perfect pairing, and Tsuji showed in [Tsu99] that in fact we have a perfect pairing (in good situations) in the limit, i.e. for log-crystalline cohomology. With that motivation, I have been studying with Prof. Chiarellotto whether one can find a duality between the kernel of the monodromy (and hence rigid cohomology) with the cokernel of the monodromy operator on the double complex. This question is closely related to a question that I am working on alongside Prof. Chiarellotto and other collaborators about a special case on a conjecture of Flach and Morin. At the moment I am looking at potential sources of the appropriate duality, in particular extending the complex-analytic duality results in [ElZ83] to the p-adic setting. Currently we are investigating whether we can use the equivalence of log-rigid cohomology (as defined by Große-Klönne) and log-crystalline cohomology in the proper setting along with a rigid-analytic equivalent of El Zein's constructions to transport El Zein's complex-analytic duality to the rigid-analytic setting.

I have also recently started working with my coadvisor Prof. Christian Liedtke at the Technische Universität München on extending the techniques of [LM18] to the case of semi-stable reduction by first applying the technique to prove the potential semi-stable reduction (after an unramified extension) of elliptic curves, a result which is by now classical. The idea is to use the techniques used to prove the potential good reduction of K3 surfaces after an unramified extension to the proof of potential *semi-stable* reduction of *elliptic curves* after an unramified extension, and then in turn to apply this technique to prove the potential semi-stable reduction of *K3 surfaces* after an unramified extension. I am currently in the early stages of this research, studying background material on various models of elliptic curves over a Henselian DVR, and a different proof of the potential good reduction of elliptic curves via a study of the special fibers of the Nèron models and minimal models of elliptic curves.

References

[Chi99] Bruno Chiarellotto. “Rigid Cohomology And Invariant Cycles for A Semistable Log Scheme”. In: *Duke Mathematical Journal* 97.1 (1999), pp. 155–169.

[ElZ83] Fouad ElZein. “Mixed Hodge Structures”. In: *Transactions of the American Mathematical Society* 275.1 (1983), pp. 71–106.

[Har66] R. Hartshorne. *Residues and Duality*. Springer, 1966.

[LM18] Christian Liedtke and Yuya Matsumoto. “Good reduction of K3 surfaces”. In: *Compositio Mathematica* 154.1 (2018), pp. 1–35. arXiv: 1411.4797.

[Mok93] A. Mokrane. “La suite spectrale des poids en cohomologie de Hyodo-Kato”. In: *Duke Math. J.* 72.2 (Nov. 1993), pp. 301–337.

[Tsu99] Takeshi Tsuji. “Poincare Duality for Logarithmic Crystalline Cohomology”. In: *Compositio Mathematica* 118 (1999), pp. 11–41.