

# FINAL REPORT

## Dates

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**Name of the thesis:** The computational Content of Abstract Algebra and  
Analysis

## Research Fields

At the beginning of my PhD study I have considered the signed digit representation of real numbers. I developed an algorithm which takes a sequence of reals and gives back its limit if it exists. To get this algorithm, I used the method of program extraction and the proof assistant Minlog. My results were submitted in a paper, which is currently under review. In the 1st Swiss-Italian Workshop on Proof and Computation and ABM 2018 I gave a talk about my work on the signed digit streams. From 01.06.2018 to 24.08.2018 I have visited the seminars at the Hausdorff Institute in Bonn, where I have met many people who are working in the same research field as I do.

My first task in the second year was to study some papers written by Thomas Powell. One of them is about an extension of Gödel's Functional interpretation by using states [27]. I implemented some parts of this in the proof assistant Minlog and also gave a talk about it at the Münchenwiler Meeting 2018. After that I had a closer collaboration with Thomas Powell and we used some of his results [25, 26, 24] as a foundation to produce an algorithm which approximates ideal objects in algebra. We published these results and some applications in the anthology "Logic, Languages, Information and Computation" of the "Workshop on Logic, Languages, Information and Computation". At this workshop I gave a talk about this publication at the beginning of July. We have also worked on further applications of this algorithm based on [28]. Hereby I have got a constructive interpretation of the Theorem of Gauß-Joyal, Theorem of Kroneker and Dedekind's Prague Theorem. Another main goal of mine was to develop a constructive interpretation of Artin's proof of Hilbert's 17<sup>th</sup> problem which is a part of

constructive real algebra. Hereby I studied several papers and books like [4, 11, 12, 18, 20, 29] and gave a talk about the state of my research at the Conference “Algebra and Algorithm” in Djerba. However, after months of studying it turned out that this topic is either unsolvable or way to much for a three-year PhD study. Therefore, I closed this topic at the beginning of 2020 without any publishable results.

In the beginning of 2019 I stated my cotutelle with the Ludwig-Maximilians University (LMU) in Munich. Here I have worked together with Prof. Schwichtenberg, who is my cotutelle supervisor at LMU, and we have written the paper “Logic for exact real arithmetic”. At LMU I have also given a seminar talk about continuous functions in constructive analysis.

From the 20<sup>th</sup> of January to the 19<sup>th</sup> of February 2020 I have visited Thomas Powell at the technical university in Darmstadt where we have considered proofmining in analysis. A detailed report of my stay in Darmstadt is given in the document “Visit to TU Darmstadt”. During my stay in Darmstadt I attended the Conference “Algebra and Algorithm” in Djerba and participated in the soft skill seminar “Redefabrik CarismaCore”, which was a one-day seminar about rhetoric and charisma given by Benedikt Held in Frankfurt.

During my hole PhD study I read many books like [14, 16, 17, 19, 23, 31] about rhetoric, psychology and communication to improve my presentation and social skills.

Since the Corona lockdown in March I have mainly worked from home. Unfortunately it was not possible to visited Thomas Powell a second time and to finish our paper but we have stayed in contact via Skype and we are currently still working on this paper.

During the lock down I have attended the online seminar “Meisterkurs Rhetorik Mentoring” which was about rhetoric and holding lectures [15]. It took place on six successive Mondays starting with the 23<sup>rd</sup> of March.

Furthermore, I have developed a constructive proof of Zariski’s Lemma. Hereby I had a look at the sources [5, 6, 21, 30, 32, 34] and used them as inspiration to establish a direct and constructive proof of Zariski’s Lemma. Out of this constructive proof I developed an algorithmic version of Zariski’s lemma. I have written down the results of this research in the paper “An algorithmic version of Zarsiki’s lemma”, which I have submitted for Types 2020.

I am going to finish my PhD study at the end of January 2021 by finishing my PhD thesis and writing the the paper about proofmining in analysis together with Thomas Powell.

## Courses at the University of Trento

- **Homological Algebra** given by Edoardo Ballico  
As the exam I wrote a paper about Mitchell's embedding Theorem.
- **Algebraic Geometry II** given by Sola Conde  
As the exam I gave a talk about the Ax-Grothendick Theorem.
- **Model Theory** given by Stefano Baratella  
As the exam I gave a talk about saturated models.

## Conferences, Seminars and Workshops

- **ABM 2017** in Munich from 14.12.2017 to 15.12.2017 <http://www.math.lmu.de/~petrakis/ABM17.php>
- **1st Swiss-Italian Workshop on Proof and Computation** in Verona from 18.01.2018 to 19.01.2018 <https://logicseminarverona.wordpress.com/2017/11/30/save-the-date-2/>  
Given talk: *Limit Values in the Signed Digit Representation*
- **Computational Approaches to the Foundations of Mathematics** in Munich from 11.04.2018 to 13.04.2018
- **ABM 2018** in Bern from 26.04.2018 to 27.04.2018 <http://abm.inf.unibe.ch/organization.html>  
Given talk: *Limits in the Signed Digit Representation of Reals*
- **Types, Homotopy Type theory, and Verification** at the Hausdorff Research Institute for Mathematics in Bonn from 04.06.2018 to 08.06.2018 <https://www.him.uni-bonn.de/programs/past-programs/past-trimester-programs/types-sets-constructions/workshop-types-homotopy-ty>
- **Proofs and Computation** at the Hausdorff Research Institute for Mathematics in Bonn from 02.07.2018 to 06.07.2018 <https://www.him.uni-bonn.de/programs/past-programs/past-trimester-programs/types-sets-constructions/workshop-proofs-and-computation/>
- **Proof, Computation, Complexity** at the Hausdorff Research Institute for Mathematics in Bonn from 19.07.2018 to 20.07.2018 <https://www.him.uni-bonn.de/programs/past-programs/past-trimester-programs/types-sets-constructions/workshop-proof-computation-complexity/>

- **Constructive Mathematics** in Hausdorff Research Institute for Mathematics in Bonn from 06.08.2018 to 10.08.2018 <https://www.him.uni-bonn.de/programs/past-programs/past-trimester-programs/types-sets-constructions/workshop-constructive-mathematics/>
- **Autumn School “Proof and Computation”** in Fischbachau from 16.09.2018 to 22.09.2018 <http://resources.illc.uva.nl/LogicList/newsitem.php?id=8906>
- **Münchenwiler Meeting Autumn 2018** in Münchenwiler from 24.10.2018 to 25.10.2018 <http://mw.inf.unibe.ch/>  
Given talk: *Applications of the Functional Interpretation with States*
- **ABM Sping 2019** in Munich from 02.05.2019 to 03.05.2019 <http://cj-xu.github.io/abm19/index.html>  
Given talk: *An Algorithmic Approach for Maximal Objects in Algebra*
- **26<sup>th</sup> Workshop on Logic, Languages, Information and Computation** in Utrecht from 02.07.2019 to 05.07.2018 <https://wollc2019.sites.uu.nl/>  
Given talk: *An algorithmic approach to the existence of ideal objects in commutative algebra*
- **Autumn School “Proof and Computation”** in Herrsching from 20.09.2019 to 26.09.2019 <http://www.mathematik.uni-muenchen.de/~schwicht/pc19.php>  
Given talk: *Introduction to the proof assistant Minlog*
- Talk given on 07.01.2019 at the seminar **Konstruktive Analysis** (english: Constructive Analysis) at the LMU with the title **Stetige Funktionen** (english: Continuous Functions) <http://www.mathematik.uni-muenchen.de/~schwicht/seminars/semws18/index.php>
- **Logik-Arbeitstagung München, Bern und Verona** in Munich from 12.12.2019 to 13.12.2019 <http://www.mathematik.uni-muenchen.de/~wiesnet/ABMV2019.php>  
Organiser
- **Visit to the TU Darmstadt** in Darmstadt from 20.01.2020 to 19.02.2019
- **Redefabrik CharismaCore** in Frankfurt, 25.01.2020
- **Conference “Algebra and Algorithms”** in Djerba, Tunisia from 04.02.2020 to 06.02.2020 <https://sites.google.com/view/algebraalgorithms2020>  
Given talk: *Constructive Real Algebra*

- **Meisterkurs Rhetorik Mentoring**, Online Seminar about rhetoric in six episodes from 23.03.2020 to 27.04.2020

## Publications

- **Introduction to Minlog** published by World Scientific in the volume "Proof and Computation"; Editors: Klaus Mainzer, Peter Schuster and Helmut Schwichtenberg; July 2018. <https://www.worldscientific.com/worldscibooks/10.1142/11005>
- Under review: **Limits with Signed Digit Streams**
- **An algorithmic approach to the existence of ideal objects in commutative algebra** published by Springer in the volume "Logic, Languages, Information and Computation"; Editors: Iemhoff, Rosalie, Moortgat, Michael, de Queiroz, Ruy (Eds.) ; July 2019. <https://www.springer.com/gp/book/9783662595329>
- Under review: **Logic for exact real arithmetic** <https://arxiv.org/abs/1904.12763>
- **An algorithmic version of Zariski's lemma** submitted for TYPES 2020 post-proceedings: <https://types2020.di.unito.it/>
- Under review: **A universal algorithm for Krull's theorem**

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