Report of the academic activity during the second year of the PhD program

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1. Courses and seminars

I followed the online courses/seminars:

- (Assisted) *Homological Projective Duality*,
  Prof. Daniel Huybrechts, University of Bonn.
  `http://www.math.uni-bonn.de/~huybrech/SeminarHPD.pdf`

- (Assisted) *The moduli space of curves*,
  Postdoc Johannes Schmitt, University of Bonn.
  `https://www.math.uni-bonn.de/people/schmitt/moduli_of_curves`

Also, I followed the series of seminars Topics in Mathematics and other seminars organized by the department, in particular the ones organized by the geometry and algebra group. Furthermore, I was invited to the following seminars to give a couple of lectures,

- (Invited) *Symplectic automorphisms of K3 surfaces*, Seminar of algebraic geometry organized by Prof. Pedro Montero, and Prof. Jenia Tevelev., at *Universidad Catolica de Chile*
  `https://youtu.be/2efdYBXTm-Y`

- (Invited) *Automorphisms on K3 surfaces*, Seminar of algebraic geometry organized by the research group of algebraic and physics mathematics, at SISSA.
  `https://math.sissa.it/seminar/lectures-k3-surfaces`

2. Conferences and schools

- *Fourth Latin American School on Algebraic Geometry and its Applications*, *Universidad de Talca*, Chile, December 2-13, 2019.

- (Organizer with Prof. Giovanni Mongardi) *Workshop EXARCHOS*, *Università di Bologna* at Ravenna, January 22-24, 2020. `https://eventi.unibo.it/exarchos`

  I also followed several online conferences related with my research topic in the PhD. Some of them are:

- *Japanese-European Symposium on Symplectic Varieties and Moduli Spaces* `http://www.riess-web.com/math/JES/`

- *Kickoff Meeting* del *ERC Synergy Grant* `https://www.erc-hyperk.org/`

4. Research activity

K3 surfaces that admit a symplectic automorphism of order three

(in collaboration with prof. Alice Garbagnati at Milano)

K3 surfaces have special topological properties, some of them derived from Hodge Theory and Classical Lattice Theory. Nikulin in [Nik79] proves that the action of symplectic groups in the integer second group of cohomology for a K3 surface, which is isometric to the direct sum of three copies of the hyperbolic plane and two copies of the integer lattice $E_8(-1)$, is unique up to isometries. The first to describe the action using Nikulin's results is Morrison in [Mor84], where he shows that for a group generated by a symplectic involution, the action respects the abstract decomposition before, and therefore acts by switch the two copies of $E_8(-1)$. In the work of [VS07], they parameterize the 11-dimensional irreducible components of the moduli space of algebraic K3 surfaces with a symplectic involution, and give examples of the general K3 surfaces in the different components. However, in the case of other orders, the action does not respect the abstract decomposition, even if it is unique. That allows to complement the classical way to resolve this kind of problems by implement other techniques, as the ones comes to the moduli of elliptic curves, and so one the particular case of K3 surfaces with elliptic fibrations.

In the work of [GS07], they describe the action in the cases where the automorphism has prime order, i.e., when the symplectic group has order $3, 5$, and 7. In this project, we want to parameterize the irreducible components of the moduli space of algebraic K3 surfaces that admits a symplectic automorphism of order

three. This result has very interesting implications in the study of K3 surfaces that have a Shioda-Inose structure. Hence, the second part of the project would be to give some criteria regarding the questions on K3 surfaces that admit such structures. We are working in the first preprint version.

Symplectic automorphisms of Hyperkähler manifolds

A well-known fact is that Hyperkähler manifolds can be thought as higher dimensional analogs of K3 surfaces. In this part, we want to generalize some results of the previous project for some Hyperkähler manifolds that admit symplectic automorphisms, as the example of Namikawa [Nam01]. The technique used in the case of surfaces is almost the same in the case of higher dimension but the obstructions are many, in particular those that came from the description of the symplectic group action in the integer second group of cohomology. What should be done is to study the intermediate variety obtained by doing the quotient through a symplectic group. These varieties are singular Hyperkähler manifolds and they are of recent development in the field. Some examples are described in [CGM19].

5. Further research and stays

None because of the global health emergency.

6. Future plans

If the global health situation allows, I will go for a period of three months at the mathematics institute of the University of Bonn with Prof. Daniel Huybrechts.

References

[Nik79]   V. V. Nikulin. "Finite groups of automorphisms of Kählerian $K3$ surfaces". In: *Trudy Moskov. Mat. Obshch.* 38 (1979), pp. 75–137. ISSN: 0134-8663.

[Mor84]   D. R. Morrison. "On $K3$ surfaces with large Picard number". In: *Invent. Math.* 75.1 (1984), pp. 105–121. ISSN: 0020-9910. DOI: 10.1007/BF01403093. URL: https://doi.org/10.1007/BF01403093.

[Nam01]   Yoshinori Namikawa. "Deformation theory of singular symplectic n-folds". In: *Math. Ann.* 319.3 (2001), pp. 597–623. ISSN: 0025-5831. DOI: 10.1007/PL00004451. URL: https://doi.org/10.1007/PL00004451.

[GS07]   Alice Garbagnati and Alessandra Sarti. "Symplectic automorphisms of prime order on $K3$ surfaces". In: *J. Algebra* 318.1 (2007), pp. 323–350. ISSN: 0021-8693. DOI: `10.1016/j.jalgebra.2007.04.017`. URL: `https://doi.org/10.1016/j.jalgebra.2007.04.017`.

[VS07]   Bert Van Geemen and Alessandra Sarti. "Nikulin involutions on $K3$ surfaces". In: *Math. Z.* 255.4 (2007), pp. 731–753. ISSN: 0025-5874. DOI: `10.1007/s00209-006-0047-6`. URL: `https://doi.org/10.1007/s00209-006-0047-6`.

[CGM19]   Chiara Camere, Alice Garbagnati, and Giovanni Mongardi. "Calabi-Yau quotients of hyperkähler four-folds". In: *Canad. J. Math.* 71.1 (2019), pp. 45–92. ISSN: 0008-414X. DOI: `10.4153/cjm-2018-025-1`. URL: `https://doi.org/10.4153/cjm-2018-025-1`.