INdAM-DP-COFUND-2015
(Final report on research activities)

Dear President of INdAM,

My name is Nhu Ngoc Nguyen, I am a Ph.D. student at the Department of Mathematics, Politecnico di Milano, Italy. I would like to present my final report on the research work during the fellowship. This report includes a summary of main research progress, achievements and activities carried out under the project **"INdAM Doctoral Programme in Mathematics and/or Applications Cofunded by Marie Skłodowska-Curie Actions"** by the **Istituto Nazionale di Alta Matematica**, acronym: INdAM-DP-COFUND-2015, grant number: 713485.

On 23 October 2020, the Doctoral Committee of the Department of Mathematics (Politecnico di Milano) met for the annual evaluation of Ph.D. candidates. The board of professors admitted me for the final defense with a passing grade A.

1 General information

Affiliate: Dipartimento di Matematica "Francesco Brioschi", Politecnico di Milano
Cycle: XXXIII
Ph.D. course: Mathematics Models and Methods in Engineering
Supervisor: Professor Silvia Lorenzani
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2 Research project

2.1 Research project title

Variational solutions of the linearized Boltzmann equation for thermal – driven gas flows in microchannels.

2.2 Project progress and achievements

My Ph.D research project concerns the application of kinetic theory to the study of microdevices. The micromachinery fabrication techniques have become more and more mature in the last few decades. In particular, the micro-electro-mechanical system (MEMS) devices (inertial sensors, accelerometers, gyroscopes) can be found in many applications such as microelectronics, medicine, biology, optics and other high technology fields.

Micro- and nanodevices are often operated in gaseous environments (typically air), and thus their performances are affected by the gas around them. Since the smallest characteristic length of MEMS is comparable with (or smaller than) the mean free path of the gas molecules, the traditional computational fluid dynamics methods, based on the Euler or the Navier-Stokes equations, fail in predicting the flows related to these devices. Therefore, an accurate analysis of such microfluidic systems requires the solution of the Boltzmann equation (BE), which describes the evolution of the velocity distribution function of the gas molecules in non-equilibrium statistical mechanics. In spite of the apparently complex structure, the basic constituent of a real MEMS device is the microchannel, the region between two parallel plates that can reveal many specific features of the low speed internal flows in microdevices.

Therefore, an important aspect of the matter is to have an approximate closed form solution for gas flow rates in microchannels in order to use it in applications. In order to develop an accurate formula directly from kinetic theory, there is a particularly useful technique: the variational method proposed by C. Cercignani in 1969 [C. Cercignani, J. Stat. Phys. 1, 297 (1969)]. We stress that this variational technique applies directly to the integrodifferential form of the BE and can be used for any linearized Boltzmann equation, while the variational methods used by other authors apply to the integral form of the BE, which is available only for special models, such as the Bhatnagar, Gross, and Krook (BGK) model.

In the current project, the variational technique is used to compute temperature-driven (thermal creep) gas flows in microchannels by considering the true linearized Boltzmann collision operator. We have paid particular attention to the role of boundary conditions (which describe the interaction of the gas molecules with the solid walls) by investigating different kinds of models of gas-surface interactions.

Accurate modeling of gas-surface interaction is very important for external rarefied gas flows, such as those around shuttles and satellites, as well as for internal rarefied or small-scale flows, like those in MEMS or in gas shale reservoirs, both characterized by large surface area to volume ratios. The rarefaction level of a gas can be quantified by the rarefaction parameter, i.e. the ratio between a characteristic dimension of a flow and the molecular mean free path of a gas. The most reliable description of rarefied gas flows is provided by the BE. The boundary conditions for the BE must be formulated on a probabilistic ground, by specifying the relation between the velocity distribution function of the reflected and the incident gas molecules on the solid surface, the so-called scattering kernel. In the case of low rarefaction level, the gas flow can be still simulated in the frame of the continuum modeling based on the Navier-Stokes-Fourier system or on higher-order expansions as the 13 moment equations (R13). However, the implementation of special boundary conditions, like the velocity slip and temperature jump on gas-solid interfaces, is necessary to take into account the rarefaction effects. All these conditions involve the viscous-slip, the thermal-slip and the temperature-jump coefficients, which depend on the gas-solid surface interaction through the accommodation coefficients. Different scattering kernels can be found in the literature: Maxwell, Cercignani-Lampis (CL), Epstein, Klinc & Kuščer, Dadzie-Méolans and some others. These proposed models have one or several adjustable parameters that can be associated with the accommodation processes of physical quantities. For example, the Maxwell model contains only one parameter, which is the accommodation coefficient of all molecular properties. While the Cercignani-Lampis model includes two parameters: the accommodation coefficient of the tangential momentum, and the accommodation coefficient of the kinetic energy due to the normal velocity.

One of the main objectives of this work is to improve the fundamental understanding of the gas-surface interaction. To this end, the variational approach has been used to obtain asymptotic near-continuum solutions of the linearized Boltzmann equation for hard-sphere molecules, in order to provide analytical expressions for the first- and second-order thermal slip coefficients.

A further goal of the present investigation is to interpret recent experimental data on thermal creep flows, in a wide range of the rarefaction parameter, useful in many MEMS applications. I will give a summary of the results that have been obtained from our work in the following sections.

1. The Bhatnagar, Gross and Krook (BGK) model

In general, it is difficult to manage the Boltzmann collision operator as such. Therefore, simplified kinetic models of the exact collision integral are widely used in practice, for both analytical calculations and numerical simulations. Because of its simplicity, the BGK model is one of the most popular of these kinetic models. Therefore, as first approach, we use the variational technique applied to the Boltzmann equation, based on the BGK model, in order to compute the thermal creep flow rate as a function of the Knudsen number (defined by the ratio of the molecular mean free path to a characteristic length of the system). The variational results have been compared with accurate numerical solutions of the BGK-Boltzmann equation.

1.1 The thermal-creep problem (full solution for the heat-flow with complete diffusion boundary conditions)

The heat flow rate in the thermal creep problem has been rigorously evaluated by means of a variational technique which applies to the integrodifferential form of the Boltzmann equation based on the BGK model. We have assumed the simplest form of the gas-wall boundary conditions, that is the complete diffusion model according to which the gas molecules are reemitted by the channel plates with the Maxwellian distribution of the wall. A comparison between our variational results and the numerical outputs available in the literature [Sharipov et al., Phys. Fluids 21, 067101 (2009)] gives a fairly good agreement (the relative error is less than 0.1%) over the whole range of Knudsen numbers. It should be pointed out that the variational method used by other authors applies to the integral form of the Boltzmann equation, and therefore it requires the evaluation of cumbersome integrals which involve various analytical and numerical approximations in order to be derived and computed.

1.2 Coupled Poiseuille and thermal-creep problems

The variational technique, applied to the BGK-Boltzmann equation, is used to compute both Poiseuille and thermal-creep flows by considering the complete diffusion and Cercignani-Lampis scattering kernel for the gas-surface interaction.

1.2.1 Maxwell diffusion boundary conditions

In order to evaluate the temperature-driven mass flow rate, our variational technique is used to solve in a unified manner the Poiseuille and thermal-creep problems by means of the computation of only one functional. The complete diffusion model of boundary conditions has been considered. The agreement between the outcomes of our variational method and the findings obtained in previous numerical works [Sharipov et al., Phys. Fluids 21, 067101 (2009)] is very good (the relative error is less than 0.2%).

1.2.2 Cercignani-Lampis (CL) boundary conditions

An advantage of the variational technique is that it allows us to write down explicit approximate equations to be used in practical design by suitable analytical manipulations. In particular, the asymptotic near-continuum solutions of the BGK model equation for the Poiseuille and thermal-creep problems have been used in order to obtain an analytic expression for the first- and second-order slip coefficients. The purpose of our investigation is to modify the boundary conditions for the Navier-Stokes equations to extend their applicability to the slip and transitional regimes. Moreover, in order to take into account the influence of the accommodation coefficients on the slip parameters, we have considered the linearized BGK-Boltzmann equation and the CL scattering kernel. We have obtained a value of the first-order thermal-slip coefficient in close agreement with the results published by Siewert et al. [Phys. Fluids 14, 4123 (2002)] for all values of the accommodation coefficients.

2. The true linearized Boltzmann equation for hard-sphere molecules

Although the BGK model is one of the most popular kinetic models, it is known to have a serious flaw: it leads to a wrong Prandtl number (i.e. the dimensionless ratio of viscosity and thermal conductivity). This difficulty can be dealt with, when one works in the linearized framework, since viscosity and temperature effects are then decoupled and either viscosity or thermal conductivity can be fixed to a correct value. However, if one wants to analyze, at the same time, the Poiseuille and thermal-creep problems, where the mass and heat fluxes are due to the combined effects of the pressure and temperature gradients, the BGK model can not provide simultaneously accurate results. Thus, one can resort to a more refined kinetic model such as the one proposed by Shakhov (S-model). But still, the S-model does not include any information about the interaction potential between the gas molecules, while some phenomena related to the thermal-creep problem are very sensitive to the intermolecular force laws. Therefore, in the current investigation, we first consider the thermal-creep problem to obtain the heat-flow, then we analyze the coupled Poiseuille and thermal-creep flows, on the basis of the exact linearized Boltzmann collision operator for hard-sphere molecules, in order to obtain a better approximation of real-gas behaviors.

2.1 The thermal-creep problem (full solution for the heat-flow with complete diffusion boundary conditions)

The variational approach has been used to solve the thermal-creep problem between two parallel plates as an issue of relevance for applications. The variational method turns out to be accurate in evaluating the heat flow rate progressing from free molecular, through transitional, to continuum regions. Since the implementation of our variational solution is computationally straightforward, an accurate database can be created for the gas flow rates at different Knudsen numbers. In this investigation, we focus upon the complete diffusion, according to which gas molecules are diffused with the Maxwellian distribution of the wall. We must emphasize that we have considered the trial function introduced for the BGK model and then, after some analytical manipulations, we have performed a separation of variables in phase space, in order to get an expression easy to handle when the true Boltzmann collision operator is used. The agreement between our variational results and the numerical findings published by Sharipov et al. [Phys. Fluids 21, 067101 (2009)] is fairly good in the free-molecular regime (the relative error is less than 6.7%), while the discrepancy slightly increases in the transition region and decreases in the near-continuum regime with the relative error decreasing from 12.9% to 9.7%.

2.2 Coupled Poiseuille and thermal-creep problems (asymptotic solution)

We have investigated the Poiseuille and thermal-creep problems of rarefied gas in the microchannel, by means of a variational technique which applies to the integrodifferential form of the Boltzmann equation, based on the true linearized collision operator for hard-sphere molecules and two models of boundary conditions: the Maxwell and the Cercignani-Lampis (CL) scattering kernel of the gas-surface interactions. The variational method is a powerful technique for finding approximate closed-form solutions to the Boltzmann equation. In particular, this approach has proved useful in computing parameters to be employed in classical hydrodynamical equations, when low working pressures impose corrections due to gas rarefaction effects. A typical example is the derivation of slip coefficients needed to modify the boundary conditions associated with the Navier-Stokes equations. Therefore, we have considered the near-continuum solution for the thermal creep mass flux to obtain an analytic expression for the first- and second-order slip coefficients.

2.2.1 Maxwell boundary conditions

The Maxwell gas-surface interaction law is the most widely used in the literature due to its simplicity. This model is characterized by only one accommodation coefficient for all molecular properties. Therefore, in general, one can associate the accommodation either with the accommodation of the tangential momentum or with the accommodation of energy. In particular, the Maxwell model has been successfully applied to describe various isothermal flows driven by a pressure gradient. When the temperature of the system is kept constant, the momentum exchange mechanism assumes a dominant role and the accommodation coefficient can be identified with the tangential momentum accommodation coefficient (TMAC). In a different situation, when only an exchange of energy takes place between a gas and a surface, without the macroscopic gas motion, the thermal or energy accommodation coefficient is introduced in the frame of the Maxwell model, to characterize the peculiarity of this type of interaction.

The diffuse-specular reflection condition of Maxwell's type has been considered in order to take into account the influence of the accommodation coefficient on the slip parameters. The results of the first-order thermal slip coefficient for several values of the accommodation coefficient have been compared with those reported by Siewert [Phys. Fluids 15, 1696 (2003)] and Sharipov [J. Phys. Chem. Ref. Data 40, 023101 (2011)], inferred from a strictly numerical solution of the linearized Boltzmann equation for hard-spheres. The agreement between our results and the numerical outputs is fairly good (the relative error is less than 3.5%) when the accommodation coefficient is smaller than 0.6. For bigger, a deviation between the variational and the numerical data slightly increases (the relative error is 9.2% for the complete accommodation).

2.2.2 Cercignani-Lampis (CL) boundary conditions

Most of the works in rarefied gas dynamics are based on the implementation of the classical Maxwell gas-surface interaction law, characterized by a single accommodation coefficient, while in practice, every physical quantity (i.e. momentum and energy) should have its own accommodation coefficient. In the literature, there are many data related to rarefied gas flows between two parallel plates that can be accurately reproduced on the basis of the Boltzmann equation and the Maxwell model of boundary conditions. This is especially true for isothermal gas flows. However, recent measurements of the thermal creep flow through microchannels have revealed that it is very difficult to extract from the experimental data only one accommodation coefficient, which allows one to describe all flow properties. Therefore, in order to include a more realistic physical description of the gas-surface interaction, in the current investigation, we have focused on the CL scattering kernel. This model is based on two different adjustable parameters: the accommodation coefficient of the tangential momentum and the accommodation of the kinetic energy owing to the velocity normal to the bounding walls. In this case, the variational method technique allows us to obtain an explicit relation between the first- and second-order thermal slip coefficients and the tangential momentum and normal energy accommodation coefficients, defined in the frame of the CL scattering kernel. The variational outputs on the first-order thermal slip coefficient overstate those derived from the numerical solution of the linearized Boltzmann equation reported by Siewert [Phys. Fluids 15, 1696 (2003)] (the relative error lies within 10%) and understate the findings obtained through the S-model published by Sharipov [Eur. J. Mech. B/Fluids 22, 133 (2003)] and Siewert [Phys. Fluids 14, 4123 (2002)] (the relative error lies within 5%), for each value of the accommodation coefficients. The better agreement between the variational results and those obtained on the basis of the S- model reported by Siewert [Phys. Fluids 15, 1696 (2003)] and Sharipov [Eur. J. Mech. B/Fluids 22, 133 (2003)] is likely related to the different numerical approximations involved in solving the true linearized Boltzmann equation and a simplified kinetic model. It is worth noting that in his review paper, Sharipov [J. Phys. Chem. Ref. Data 40, 023101 (2011)] recommends to use, for practical calculations in the case of diffuse re-emission, the following value of the thermal-slip coefficient $\sigma = 1.1$, which is identical to the value obtained through our variational computations.

3. **Comparison with experimental data**
Our results based on the true linearized Boltzmann collision operator for hard-sphere molecules and the Cercignani-Lampis (CL) scattering kernel have been compared with the experimental data reported by Yamaguchi et al., J. Fluid Mech., 795, 690 (2016) for five different noble gases (Helium, Neon, Argon, Krypton and Xenon) and, for each of them, a pair of accommodation coefficients has been extracted. Then, these values have been used to compute, by means our variational technique, the temperature-driven mass flow rates and the outputs have been compared with the measurements for Helium, Neon and Argon. The good agreement obtained between the theoretical and the experimental data, within the range of validity of the proposed second-order slip model, suggests that the CL boundary conditions, unlike the Maxwell model, can conveniently be used to describe non-isothermal gas flows. Instead, Wu et al., J. Fluid Mech., 823, 511 (2017) came to the opposite conclusion relying on a comparison of their numerical results with the same experimental data. However, the approach used by Wu et al. to find a pair of accommodation coefficients is different from the method applied in the present study. In their work, the accommodation coefficients have been selected to provide the best agreement between the mass flow rate measurements of thermal creep flow and the numerical calculations, as the rarefaction parameter (inverse Knudsen number) changes. Indeed, Wu et al. were unable to obtain values of the accommodation coefficients that could provide a good agreement with the experimental data of the mass flow rate over a wide range of gas rarefaction. Of course, when a model with more than one free parameter is considered, there may be different combinations of the coefficients that produce equally good results, at least locally. The advantage of the method proposed in the present investigation is that the two accommodation coefficients are preliminarily determined by imposing that the first- and second-order thermal-slip coefficients theoretically derived can closely reproduce the experimental data. These two constraints allowed us to uniquely determine two accommodation coefficients, and indeed the match, that we have obtained with the measured values of the temperature-driven mass flow rates for Helium, Neon, and Argon, is quite good even beyond the range of validity of the proposed second-order slip model. For all the gases analyzed, the tangential accommodation coefficient is found to be much larger than the normal energy coefficient. The general trend, according to which, by increasing the molecular weight of the different gases, the values of both accommodation coefficients also increase, is confirmed in this study.

3 List of educational activities carried out during the fellowship

I attended 6 courses in total. The list of courses is listed below.

3.1 Courses from Department of Mathematics

1. **Semilinear elliptic equations** by Prof.Verzini Gianmaria, and Prof. Soave Nicola from 08 Nov 2017 to 31 Jan 2018. It took place at the Department of Mathematics, Politecnico di Milano. (**30 cum laude**)

2. **Principle of maximum and applications to elliptical and parabolic differential equation of the second order** by Prof. Monticelli Dario Daniele, and Prof. Punzo Fabio from 01 Feb to 31 Mar 2018. It took place at the Department of Mathematics, Politecnico di Milano. (**30 cum laude**)

3. **Semigroups of linear operators and applications to evolution equations** by Prof. Conti Monica, and Prof. Pata Vittorino from 15 Mar to 30 May 2018. It took place at the Department of Mathematics, Politecnico di Milano. (**29/30**)

3.2 Ph.D. school courses

1. **Scientific communication in English** by Prof. Biscari Paolo, and Prof. Sluckin Timothy from Jan 16 to 30 Apr 2018. This course was held at Politecnico di Milano. (**28/30**)

2. **Epistemology of scientific and technical research** from 16 Feb to 02 Mar 2018. Lecturers: Prof. Michela Massimi (University of Edinburgh), Martin Kusch (University of Vienna), Patrick Greenough (University of St Andrews), and Prof. Sabine Roeser (TU Delft). This course was organized by Prof. Chiodo Simona (Politecnico di Milano). (**30 cum laude**)

3.3 Winter school

Kinetic Equations describing Wave Turbulence by Prof. Juan J. L. Velázquez (University of Bonn, Germany) from 28 Nov 2018 to 1 Dec 2018. It took place at Gran Sasso Science Institute (L'aquila,Italy). (**30 cum laude**)

According to the Ph.D. program, candidates must earn 30 credits. It should be noted that I completed this request almost in the academic year 2017-2018.

4 Research activities

4.1 Recent activities

I attended several seminars, workshops, conferences, etc. Some activities are listed below.

1. **29/10/2019**: **Istituto Lombardo e Lezioni Leonardesche 2019: "ricordando i 500 anni dalla scomparsa di Leonardo da Vinci"**. This conference took place

at Palazzo di Brera (Milano), Via Brera 28 Sala Maria Teresa - Biblioteca Nazionale Braidense.

- Speaker: Prof. John David Barrow, Università di Cambridge
  Title: One Hundred Years of Universes
- Speaker: Prof. Pierre Louis Lions, Istituto Lombardo Accademia di Scienze e Lettere - Collège de France
  Title: On Mean Field Games
- Speaker: Prof. Roger Penrose, Università di Oxford
  Title: On the Power of Geometric Illustration in Mathematics and Science

2. **11/07/2019**: MOX seminar series
   Title: Preconditioning of multiphysics problems with applications to the biomechanics of the brain
   Speaker: Kent-Andre Mardal, University of Oslo and Simula Research Laboratory

3. **18/6/2019**:"Seminario Matematico e Fisico di Milano"
   Title: A new approach to quantum mechanics
   Speaker: Yakir Aharonov, Chapman University

4. **08/04/2019**: the Mid-Term Review Meeting (INdAM) with the presence of Alan Craig Head of Sector, Research Executive Agency, Marie Skłodowska-Curie COFUND.

   This event was held in Roma on April 8th (full day). I had a chance to present my background, my works, and my training experiences. In particular, I had a great opportunity to interact with other fellows.

5. **02-03/06/2019**: "Giornata INdAM 2019". It took place in Bari at the University of Bari Aldo Moro.

6. **12/03/2019**: "Seminario Matematico e Fisico di Milano"
   Title: Lanford's Theorem and the Emergence of Irreversibility
   Speaker: Jos Uffink, University of Minnesota

7. **28/11 – 1/12/2018**: (Conference) Gran Sasso Quantum Meetings @GSSI: "From Many Particle Systems to Quantum Fluids"

8. **07/07-08/07/2018**: I took part in MSCA Satellite Event ahead of the EuroScience Open Forum (ESOF) 2018 in Toulouse, France. This is one of the Marie Skłodowska-Curie Actions.

9. **22/06/2018**: "Seminario Matematico e Fisico di Milano", organized by University of Milano-Bicocca.
   Title: Vaporizing and freezing the Riemann zeta function
   Speaker: Prof. **Terence Tao**, University of California, Los Angeles.

10. **28/05/2018**: Lesson of Prof. Barry Simon, at the Politecnico di Milano.
    Title: Spectral theory, sum rules and large deviations.

11. **21/03/2018**: Seminar at the Politecnico di Milano.

    Title: Collisions and Chaos in Boltzmann-Grad limit.
    Speaker: Prof. Sergio Simonella, (ENS lione)

12. **05/02/2018**: Seminar at the Politecnico di Milano.
    Title: On the weak maximum principle in unbounded domains.
    Speaker: Prof. Italo Capuzzo-Dolcetta, Sapienza Università di Roma, GNAMPA-Istituto Nazionale di Alta Matematica

13. **13/12/2017**: Seminar at Politecnico di Milano.
    • Title: Homoenergetic solution for the Boltzmann equation. Speaker: Alessia Nota, Università di Bonn
    • Title: Validity of the Boltzmann equation: how to incorporate long-range effects into a collisional dynamics
    Speaker: Chiara Saffirio, Università di Zurigo

4.2 Suspended activities

During the third year, I had planned to attend some events, in particular, a scientific-stay in Marseille (France) was organized (3 months, at the Aix-Marseille Université) from March to July 2020. Due to the pandemic, all conferences have been postponed for a year or canceled:

1. **25 – 29 May 2020** - Politecnico di Milano (Italy). (postponed)
   Conference: The Legacy of Carlo Cercignani: from Kinetic Theory to Turbulence Modeling.

2. **7-13 June, 2020** 10th Summer school on 'Methods and models of kinetic theory', Porto Ercole (Grosseto), Italy. (postponed)

3. **13-17 July 2020** - Seoul, Korea. (postponed)
   Conference: The 32nd International Symposium on Rarefied Gas Dynamics (RGD32) will be held in Seoul, South Korea. Talk title plan: Variational solutions of the linearized Boltzmann equation for thermal – driven gas flows in microchannels.

4. **09 March - 09 June, 2020** Planning to spend 3 months in Marseille (France) at the Aix-Marseille Université. (canceled)

5 Publications

- Nhu Ngoc Nguyen, Irina Graur, Pierre Perrier, and Silvia Lorenzani *"Variational derivation of thermal slip coefficients on the basis of the Boltzmann equation for hard-sphere molecules and Cercignani–Lampis boundary conditions: Comparison with experimental results"*, Physics of Fluids, **32**, 102011 (2020). https://doi.org/10.1063/5.0025282

  In this work, a variational method is applied to solve the Boltzmann equation based on the true linearized collision operator for hard-sphere molecules and the Cercignani-Lampis boundary conditions. By using variational technique, we have obtained an explicit relation between the first- and second-order thermal slip coefficients and the tangential momentum and normal energy accommodation coefficients, defined in the frame of the Cercignani-Lampis scattering kernel. Comparing the theoretical results with the experimental data from Yamaguchi et al., J. Fluid Mech., 795, 690 (2016) has caried out.

- Vo Anh Khoa, Ekeoma Rowland Ijioma & Nguyen Nhu Ngoc, *Strong convergence of a linearization method for semi-linear elliptic equations with variable scaled production*, Computational and Applied Mathematics, **39**, 281 (2020). https://doi.org/10.1007/s40314-020-01334-0

  During the summer of 2019, I collaborated with my former colleagues (Vo Anh Khoa, University of North Carolina at Charlotte, US) and another author to present a linearization scheme applied to solve the elliptic equation of semilinear types with rapidly oscillating coefficients in a perforated cube. The homogeneous Neumann-Dirichlet boundary conditions have been considered. The results of this work have published at *Computational and Applied Mathematics*.

6 Research Group and association memberships

I am a member of the

- "Gruppo Nazionale per la Fisica Matematica" (GNFM) from 1st February 2019.
- MCAA, Marie Curie Alumni Association from 20th February 2019. (Italy and Southeast Asia Chapters)

Finally, I would like to express my sincere thanks for giving me the chance to pursue my career. Achieving Marie Skłodowska-Curie fellow of the Istituto Nazionale di Alta Matematica was a great opportunity for my research career goals. Besides opening the doors of knowledge and working with experts in my field, it gave me occasions to meet wonderful friends.

Nhu Ngoc Nguyen
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