

# Second Year Report

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A summary of my activities during the first two years of my PhD.

## 1 Educational Activity

### 1.1 Examined Courses

- *Topos Theory*, Olivia Caramello: Masters' course, University of Insubria, 2018.
- *Modal Logic*, Achille Frigeri: PhD course, University of Insubria, 2019.
- *Bayesian Computation and (Markov Chain) Monte Carlo simulation*, Antonietta Mira: PhD course, University of Insubria, 2019.
- *Nonlinear and nonstationary signal decomposition and analysis: Theoretical and numerical aspects and applications*, Antonio Cicone: PhD course, University of Insubria, 2020.

### 1.2 Other Attended Courses and Seminars

- *Lie Groups*, Laurent Lafforgue: PhD course, University of Insubria, 2018.
- *Statistical Learning Theory and Applications*, Lorenzo Rosasco and Prof. Silvia Villa: PhD course, University of Insubria, 2019.
- *First Order Methods in Computer Science*, Stefano Serra-Capizzano: PhD course, University of Lugano, 2019.
- *Bayesian Computation Day*, Antonietta Mira, Stefano Peluso; University of Lugano, 2019.
- *Cohomology of Toposes*, Olivia Caramello, Laurent Lafforgue: Masters' course, University of Insubria, 2019.

## 1.3 Teaching

I taught the exercise classes (in Italian) for the following second year undergraduate courses at the University of Insubria:

- *Geometria I*, 12 hours; course by Olivia Caramello, Davide Bianchi.
- *Algebra II*, 12 hours; course by Valerio Monti.

## 2 Research Activity

### 2.1 Publications

- *Toposes of Discrete Monoid Actions*, preprint, arXiv:1905.10277.
- *Monoid Properties as Invariants of Toposes of Monoid Actions*, joint work with Jens Hemelaer, University of Antwerp, under review for Applied Categorical Structures, arXiv:2004.10513.
- *An essential, hyperconnected, local geometric morphism that is not locally connected*, 3 page note based on joint work with Jens Hemelaer, University of Antwerp, arXiv:2009.12241.

### 2.2 Attended Conferences

Conferences listed in chronological order. Several more conference attendances were planned, but cancelled due to the pandemic situation.

- *6th Workshop on Formal Topology*, University of Birmingham, UK, 2019.
- *105th Peripatetic Seminar on Sheaves and Logic*, University of Palermo, Italy, 2019. I gave a talk entitled "Toposes of Discrete Monoid Actions" here.
- *Topology, Algebra and Categories in Logic 2019*, Université Côte d'Azur, France, 2019.
- *Category Theory 2019*, University of Edinburgh, UK, 2019.
- *Firbush Theta Meetup* (meeting of former University of Cambridge Mathematics Students), in association with the University of Edinburgh, UK, 2019. I gave a talk entitled "How to Train your Grothendieck Topos" here.
- *Symposium on Compositional Structures (SYCO 6)*, University of Leicester, UK, 2019. I gave a talk entitled "Monoid Properties as Topos Theoretic Invariants" here.
- *Topics in Category Theory School*, University of Edinburgh, UK, 2020. I gave a talk entitled "Monoid Properties as Topos Theoretic Invariants" here, similar in substance but more complete than the above.
- *MIT Categories Seminar*, online, 2020.

## 2.3 Ongoing Research

My ongoing research has two threads.

My individual work is based on investigating toposes of **topological monoid** actions, in three papers, all in progress.

The first paper is a direct extension of the themes of my preprint on discrete monoid actions. I identify properties of toposes of the form  $\text{Cont}(M, \tau)$  (the continuous actions of a monoid  $M$  equipped with a topology  $\tau$ ), characterising such toposes and identifying the extent to which the monoid may be recovered from these toposes.

One property of particular importance that emerged in that paper was that these toposes are **supercompactly generated**. Since all of the results regarding supercompactly generated toposes that I needed can be proved abstractly, they form the body of a second paper, whose results I apply in the first paper.

In order to emulate the logic-based results obtained by Caramello in her paper *Topological Galois Theory*, which is the motivation for this line of research, it is necessary to characterise the **fragment of logic supported by supercompactly generated toposes**. This is in general distinct from the well-studied regular or coherent logics, which rely on the categorical structure of finite limits, since the canonical sites for these toposes do not always have finite limits. In small examples, the theories classified by supercompactly generated toposes are remarkably combinatorial in flavour, such as the theory of connected, simple, irreflexive digraphs. Producing a formalism for this logic is the focus of the third paper.

The ultimate goal of these collected works is to lay the groundwork for **Topological Semi-Galois Theory**, which will be the analogue of Caramello's work, but in the broader context of topological monoids rather than topological groups; this is the principal goal of the third year of my PhD.

In parallel, my joint work with Jens Hemelaer involves characterising the properties of toposes of discrete monoid actions and the geometric morphisms between them, in order to provide new examples and counterexamples (such as the one which is the subject of our recent note) and to further develop the topos-theoretic understanding of monoids.

A less intensive ongoing side project, which my work with Hemelaer is contributing to, involves an investigation of the relationships between different types of geometric morphism.

## 2.4 Other Research Activity

In 2019 I spent one week of intensive research visiting the IHES in Paris, accompanied by my colleague Riccardo Zanfa and PhD advisor Olivia Caramello.

Since March 2020, I have been an active member and moderator of the online Zulip Category Theory forum, which has over one thousand active users internationally.