

Relation associated with the research activity during the second year of the Ph.D. program

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Host institution: Alma Mater Studiorum - The University of Bologna

Supervisor: Prof. Fausto Ferrari

Cycle: XXXIV

1. Attended courses and seminars

In the academic year 2019-2020 starting from the month of November I have attended (both remotely and in presence) the following courses dedicated for the Ph.D. students:

- 1) *Lie groups and lattices*,
held by Alessio Savini
Duration: 30 h.
Place: University of Bologna.
- 2) *Some Nonlocal problems in PDEs and Geometric Measure Theory*,
held by Prof. E. Cinti.
Duration: 16 h.
Place: University of Bologna.

Besides the courses stated above I took part in the cycle of seminars *Topics in Mathematics* organized by Prof. Luca Moci and Prof. Carolina Beccari that covered different branches of mathematics and adjacent subjects. Also multiple seminars held in the department and outside were frequented among which Bruno Pini analysis seminars and analysis seminars at the University of Western Australia (in remote mode).

2. Schools and workshops

- *Variational methods in nonlinear phenomena*
Place: University of Napoli (remotely)
Period: 23rd of September 2020 - 25th of September 2020
The workshop included several talks dedicated to variational analysis and partial differential equations. The organizers linked researchers that are dealing both with the classic calculus of variation and the field of nonlinear functional analysis.
- *Public forum about Maths and Covid*
Place: The University of Western Australia, Perth
Period: 27th of August 2020 (remotely)
Forum mainly covered issues related to all kinds of networks, such as networks of people or computer networks. Speakers shared mathematical models reflecting the recent pandemic Covid-19 situation.

3. Tutor activity

In the second half of the academic year 2019/2020 I was a teaching assistant for the course Mathematical Analysis T-2, held by Prof. Vittorio Martino and Prof. Fausto Ferrari on the Faculty of engineering at the University of Bologna.

4. Research activity

In the beginning of the second year of the doctorate program I continued studying the properties of the Heisenberg group including some new notions, the notion of domain variational solution as one of them. My attention was attracted by a Bernoulli functional, that has been considered by H. W. Alt, L. A. Caffarelli and A. Friedman in 1984 [1], of the form

$$F(u) = \int_{\Omega} (|\nabla_{\mathbb{H}^1} u|^2 + \chi_{\{u>0\}} + 2fu),$$

where $\Omega \subset \mathbb{H}^1$, \mathbb{H}^1 denotes the Heisenberg group, $\nabla_{\mathbb{H}^1} u$ is the horizontal gradient in the Heisenberg group, $\chi_{\{u>0\}}$ is the characteristic function of the set $\{u > 0\}$ and f is a given function. I was looking to find out the properties of minima of the above functional and to formulate correctly the two phase free boundary problem when dealing with degenerate operators in the Heisenberg group. After the fruitful collaboration with my supervisor Prof. Fausto Ferrari this ended to be an accepted work in the journal.

A large part of my scientific activity since my trip to the University of Western Australia (25/08/2019-13/10/2019) was dedicated to the study of a notion of "divergent" nonlocal operators. In [3] the authors introduce a new type of equations where the equality holds up to a possibly divergent polynomial. My goal was to discover if there are possible generalizations for a wider class of integro-differential operators, and if it is possible, then to find out the necessary conditions to put on this kind of operators. Firstly, I studied the case of the fractional Laplace operator which is essential in nonlocal framework. Due to a nonlocal nature of the equation, one has to have a regular enough function locally, and also to control the behaviour of the function at infinity. Secondly, I started exploring concrete examples of the suitable kernels. Now my task is to be sure that the imposed conditions are optimal.

In the recent times one of my research interest are fully nonlinear nonlocal equations. At the moment I am entering in the framework and study necessary definitions and the most common techniques. In the paper [2] L. Caffarelli and L. Silvestre prove the Hölder continuity of solutions to equations that involve maximal operators without using the nonlocal Harnack inequality. The natural question to me is to understand if there exist other proves of this fact. To this aim I am trying to look at it from the viscosity point of view.

5. Publications and preprints

- Dipierro, S., Dzhugan, A., Forcillo, N., Valdinoci, E. (2020). Enhanced boundary regularity of planar nonlocal minimal graphs and a butterfly effect. Bruno Pini Mathematical Analysis Seminar, 11(1), 44-67. doi:<https://doi.org/10.6092/issn.2240-2829/10585>, arxiv:1912.05794.
- Aleksandr Dzhugan, Fausto Ferrari, “Domain Variation Solutions for degenerate two phase free boundary problems”, Mathematics in Engineering, to appear, arXiv:2001.07174.

Date

09/11/2020

Signature

Aleksandr Dzhugan



References

- [1] H. W. Alt, L. A. Caffarelli, and A. Friedman. *Variational problems with two phases and their free boundaries*. Trans. Amer. Math. Soc., 282(2):431–461, 1984.
- [2] L. Caffarelli, L. Silvestre. *Regularity theory for fully nonlinear integro-differential equations*. Communications on Pure and Applied Mathematics 62 (2009): 597-638.
- [3] Serena Dipierro, Ovidiu Savin, and Enrico Valdinoci. *Definition of fractional Laplacian for functions with polynomial growth*, Rev. Mat. Iberoam.