Report of the academic activity during the second year of the PhD programme

PhD Student: Chalmoukis Nikolaos
Supervisor: Prof. Nicola Arcozzi
Tutor: Prof. Nicola Arcozzi
Circle: 33°

1. Courses and seminars

I followed the courses

- *The De Branges Theory of Hilbert Spaces of Entire Functions and its Applications to Spectral Theory of Differential Operators*
  Professor: Anton Baranov  
  Duration: 16h  
  Place: University of Bologna

- *Functional Analysis and Complex Analysis*
  Professor: Nicola Arcozzi  
  Duration: 16h  
  Place: University of Bologna

Also, I followed the series of seminars *Topics in Mathematics* and other seminars organized by the department, in particular the ones organized by the Complex Analysis Lab, the group of complex analysis of the U. of Bologna, of which I am a member. Furthermore I spent a week at the University of Thessaloniki, invited by Professor Siskakis, and I gave a talk entitled ”The Dirichlet problem on infinite trees”.

3. Conferences and Summer Schools

- *Workshop su varietà reali e complesse: Geometria, topologia e analisi armonica*
  Place: Pisa, Scuola Normale Superiore  
  Period : February 21 2019 - February 23 2019  
  The conference was the annual meeting inside the PRIN project.

- *Explorations in Harmonic Analysis and other realms*
  Place: Tel Aviv, Israel  
  Period: February 10- February 14, 2019
The aim of the conference is to cover recent developments in harmonic analysis and related areas.
http://u.math.biu.ac.il/levnir/conferences/olevskii80/

• **Analysis Days in Piemonte**  
  Place: Piemonte  
  Period: May 27 2019 - May 31 2019  
  A workshop organized by the Chebyshev Laboratory in collaboration with the department of mathematics of the University of Bologna. I gave a talk entitled ”Interpolation in the Dirichlet space on the unit disc”.  

• **Advanced courses in operator theory and complex analysis XVI edition**  
  Place: U. Paris-Est Marne-la-Valle, France  
  Period: June 17 - June 21 2019  
  A conference co-organized be the departments of mathematics of the U. of Bologna, U. Paris-Est Marne-la-Vallee, France and ICMat e U. Complutense de Madrid, Spain. I gave a talk entitled ”Onto Interpolation for the Dirichlet space and for Sobolev W1,2(D)”.

• **Indam day 2019**  
  Place: Bari, Italy  
  Period: June 3 2019

4. **Talks**

Apart from the talks mentioned in the previous paragraph, I also gave the following ones.

• **Generalized Integration operators on Hardy spaces**  
  Place: U. of Bologna  
  Date: November 10 2017  
  As part of the activities of the Complex Analysis Lab

• **Onto interpolating sequences for the Dirichlet space**  
  Place: Aristotle U. of Thessaloniki  
  Date: September 27 2018

4. **Research activity**

My general research area is Banach spaces of analytic functions and operator theory on them. In particular one of the main objects we interested in studying, my self personally and in general our research group, is the Dirichlet space $\mathcal{D}$ of analytic functions, and some weighted versions of it. The theory of the Dirichlet space is not as well developed as the corresponding theory of Hardy or Bergman spaces for example and there are major open problems concerning this class of functions, to
mention for example the Shapiro-Shields conjecture on the invariant subspaces of $D$.

In the second year of my PhD I pursued further some of the problems I consid-
ered earlier about Universal and Onto Interpolating for the Dirichlet space. The
characterization of interpolating sequences in spaces of analytic functions is an in-
teresting problem which turns out to have a variety of applications from operator
theory to control theory to mention some. In classical spaces of analytic functions
like Hardy, Bergman, Fock, at least in one variable, interpolating sequences are
quite well understood. However in Dirichlet spaces, the problem is more involved
and the known conditions are related to capacities. In this context a probabilistic
approach can lead to new insights into the problem.

In this spirit, we consider so called Steinhaus sequences, that is, random vari-
ables $\Lambda(\omega) = \{\lambda_n\}$, where $\lambda_n = r_n e^{i\theta_n(\omega)}$, where $\theta_n$ are real independent random
variables uniformly distributed in $[0, 2\pi]$, and $r_n$ is a fixed deterministic sequence.
For such sequences we study conditions on the sequence $r_n$ such that we have a
Kolmogorov 1-0 type law for interpolation. Similar problems have already been
considered for example in [2], [3] and [1]. It is interesting to notice that in fact
we get a an improvement on a previous result of Rudowicz for random Carleson
measures in the Hardy space, which is close to optimal.

Apart from the aforementioned project, in collaboration with M. Levi, we spent
also some time on developing a potential theory on acyclic connected graphs,
(known also as trees). In particular, we proved a Wiener-type criterion for regular
points for the Dirichlet problem in the tree’s boundary, a probabilistic interpreta-
tion of capacity, and some results for uniqueness of the solution of the Dirichlet
problem for harmonic functions with finite energy, i.e. belonging to some Sobolev
space.

For more information on the previous results see the papers below.

4.Papers and Preprints

- N. Chalmoukis, A. Hartmann, K. Kellay, B.D. Wick, Random interpolating
on infinite trees. Concrete Operators, 6(1), pp. 20-32.
- N. Chalmoukis, Onto Interpolation for the Dirichlet space and for $W^{1,2}(\mathbb{D})$,
2018arXiv180708193C, (Preprint)

References

[1] K. Bogdan, On the zeros of functions with finite dirichlet integral, Kodai