

Report of the academic activity during the second year of the PhD programme

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Circle: 33°

1. Courses and seminars

I followed the courses

- *The De Branges Theory of Hilbert Spaces of Entire Functions and its Applications to Spectral Theory of Differential Operators*
Professor: Anton Baranov
Duration: 16h
Place: University of Bologna
- *Functional Analysis and Complex Analysis*
Professor: Nicola Arcozzi
Duration: 16h
Place: University of Bologna

Also, I followed the series of seminars *Topics in Mathematics* and other seminars organized by the department, in particular the ones organized by the Complex Analysis Lab, the group of complex analysis of the U. of Bologna, of which I am a member. Furthermore I spent a week at the University of Thessaloniki, invited by Professor Siskakis, and I gave a talk entitled "The Dirichlet problem on infinite trees".

3. Conferences and Summer Schools

- *Workshop su varietà reali e complesse: Geometria, topologia e analisi armonica*
Place: Pisa, Scuola Normale Superiore
Period : February 21 2019 - February 23 2019
The conference was the annual meeting inside the PRIN project.
- *Explorations in Harmonic Analysis and other realms*
Place: Tel Aviv, Israel
Period: February 10- February 14, 2019

The aim of the conference is to cover recent developments in harmonic analysis and related areas.

<http://u.math.biu.ac.il/levnir/conferences/olevskii80/>

- *Analysis Days in Piemonte*

Place: Piemonte

Period: May 27 2019 - May 31 2019

A workshop organized by the Chebyshev Laboratory in collaboration with the department of mathematics of the University of Bologna. I gave a talk entitled "Interpolation in the Dirichlet space on the unit disc".

<http://math-cs.spbu.ru/wp-content/uploads/2019/02/abstracts.pdf>

- *Advanced courses in operator theory and complex analysis XVI edition*

Place: U. Paris-Est Marne-la-Valle, France

Period: June 17 - June 21 2019

A conference co-organized by the departments of mathematics of the U. of Bologna, U. Paris-Est Marne-la-Vallee, France and ICMat e U. Complutense de Madrid, Spain. I gave a talk entitled "Onto Interpolation for the Dirichlet space and for Sobolev $W_{1,2}(D)$ ".

- *Indam day 2019*

Place: Bari, Italy

Period: June 3 2019

4. Talks

Apart from the talks mentioned in the previous paragraph, I also gave the following ones.

- *Generalized Integration operators on Hardy spaces*

Place: U. of Bologna

Date: November 10 2017

As part of the activities of the Complex Analysis Lab

- *Onto interpolating sequences for the Dirichlet space*

Place: Aristotle U. of Thessaloniki

Date: September 27 2018

4. Research activity

My general research area is Banach spaces of analytic functions and operator theory on them. In particular one of the main objects we interested in studying, myself personally and in general our research group, is the Dirichlet space \mathcal{D} of analytic functions, and some weighted versions of it. The theory of the Dirichlet space is not as well developed as the corresponding theory of Hardy or Bergman spaces for example and there are major open problems concerning this class of functions, to

mention for example the the Shapiro-Shields conjecture on the invariant subspaces of \mathcal{D} .

In the second year of my PhD I pursued further some of the problems I considered earlier about Universal and Onto Interpolating for the Dirichlet space. The characterization of interpolating sequences in spaces of analytic functions is an interesting problem which turns out to have a variety of applications from operator theory to control theory to mention some. In classical spaces of analytic functions like Hardy, Bergman, Fock, at least in one variable, interpolating sequences are quite well understood. However in Dirichlet spaces, the problem is more involved and the known conditions are related to capacities. In this context a probabilistic approach can lead to new insights into the problem.

In this spirit, we consider so called Steinhaus sequences, that is, random variables $\Lambda(\omega) = \{\lambda_n\}$, where $\lambda_n = r_n e^{i\theta_n(\omega)}$, where θ_n are real independent random variables uniformly distributed in $[0, 2\pi]$, and r_n is a fixed deterministic sequence. For such sequences we study conditions on the sequence r_n such that we have a Kolmogorov 1-0 type law for interpolation. Similar problems have already been considered for example in [2], [3] and [1]. It is interesting to notice that in fact we get a an improvement on a previous result of Rudowicz for random Carleson measures in the Hardy space, which is close to optimal.

Apart from the aforementioned project, in collaboration with M. Levi, we spent also some time on developing a potential theory on acyclic connected graphs, (known also as trees). In particular, we proved a Wiener-type criterion for regular points for the Dirichlet problem in the tree's boundary, a probabilistic interpretation of capacity, and some results for uniqueness of the solution of the Dirichlet problem for harmonic functions with finite energy, i.e. belonging to some Sobolev space.

For more information on the previous results see the papers below.

4. Papers and Preprints

- N. Chalmoukis, Generalized Integration Operators on Hardy Spaces, arXiv:1909.00636, (Preprint)
- N. Chalmoukis, A. Hartmann, K. Kellay, B.D. Wick, Random interpolating sequences in Dirichlet spaces, arXiv:1904.12529, (Preprint)
- Chalmoukis, N. & Levi, M. (2019). Some remarks on the Dirichlet problem on infinite trees. *Concrete Operators*, 6(1), pp. 20-32.
- N. Chalmoukis, Onto Interpolation for the Dirichlet space and for $W^{1,2}(\mathbb{D})$, 2018arXiv180708193C, (Preprint)

References

- [1] K. BOGDAN, *On the zeros of functions with finite dirichlet integral*, Kodai Math. J., 19 (1996), pp. 7–16.

- [2] W. G. COCHRAN, *Random blaschke products*, Transactions of the American Mathematical Society, 322 (1990), pp. 731–755.
- [3] R. RUDOWICZ, *Random Sequences Interpolating with Probability One*, Bulletin of the London Mathematical Society, 26 (1994), pp. 160–164.