I started my activity as an INdAM-Marie Curie fellow at the University of Genova in October 2016. At the host institute I collaborated with Matteo Varbaro, Emanuela De Negri, Maria Evelina Rossi, and Laura Tozzo, and I also had frequent mathematical discussions with Aldo Conca. Externally I collaborated with Klaus Altmann, Matej Filip, and Anna-Lena Winz (Freie Universität Berlin). I had research meetings with Jan A. Christophersen (University of Oslo), Kevin Tucker (University of Illinois at Chicago), Claudiu Raicu (Notre Dame University), Marian Aprodu and Marius Vladoiu (University of Bucharest), Lars Kastner (Technische Universität Berlin), Bernd Sturmfels (MPI Leipzig), With Carlos Amendola (Technische Universität Munich), Dacian Bonta (nuclear cardiologist at the Emory School of Medicine). I interrupted my INdAM-Marie Curie fellowship four months earlier than expected (in June 2018) in order to accept a position as substitute professor of the Berlin Mathematical School.

The INdAM-COFUND program offered a fantastic framework for my research goals. Most importantly, it gave me the time and the freedom to explore some new research directions. Exploring a new area of mathematics takes not only time, but a calm and focused state of mind. INdAM, together with my host institute - the Mathematics Department of the University of Genova (DIMA) - provided this at the best level imaginable. I am sure that both the prestige of the position, as well as the high quality research I was able to do in these (almost) 2 years, greatly helped advance my career to the visiting professorship I currently hold at the Freie Universität Berlin.

My experience as an INdAM fellow was wonderful from all points of view. I am especially grateful to Dr. Mauro Petrucci for always answering any type of request promptly and helpfully. I also thank him for his patience with my late response in some administrative matters. To Daniela Evangelista I am thankful for making travel reimbursements so uncomplicated. DIMA was a fantastic host institute. It offered the highest level infrastructure for my project. Furthermore, local funds were made available for inviting my collaborator Klaus Altmann for a research visit. But most of all, and this is probably a unique feature of DIMA, it provided the highest levels of mathematical knowledge in a warm and friendly atmosphere.

1. Brief Description of Research Activity. As a starting point of my research activity at the University of Genova I gave talk in the Algebra and Geometry Seminar on 2.11.2017: Deformations of Monomial Ideals. The aim was to present in broad strokes to the whole department the main objectives of my INdAM-Cofund project. In the following months I gave a series of lectures on deformation theory (4 lectures, each of 2 hours). The audience was made up of local Ph.D. students, as well as some researchers interested in collaborating on the topic (Emanuela De Negri, Anjan Gupta, Matteo Penegini, Matteo Varbaro). A further talk related to DEFORMIALS at the Department of Mathematics in Genova was given by Klaus Altmann (FU Berlin), who responded to my invitation and visited Genova for a week (13-17.03.2017). All this interaction sparked three explicit projects:

1. Smoothings of Stanley-Reisner schemes (with E. De Negri and M. Varbaro - both at U Genova)
2. Deformations and the Macaulay inverse system (with L. Tozzo and M.E. Rossi - both at U Genova)
3. Versal deformations of toric singularities (with K. Altmann and M. Filip - both at FU Berlin)

My activity as an INdAM fellow included also finishing up some past projects which were published in this time.

1.1. Smoothings of Stanley-Reisner schemes. This work starts from the following question: What kind of reduced monomial schemes can be obtained as a Gröbner degeneration of a smooth projective variety? We were able to phrase an answer in several circumstances, and, most importantly, conjecture a general one: only Stanley-Reisner schemes associated toacyclic Cohen-Macaulay simplicial complexes. Our conjecture implies, in particular, that only curves of genus zero have such a degeneration. We prove this conjecture for degrevlex orders, for elliptic curves over real number fields, for boundaries of cross-polytopes, and
for leafless graphs. We discuss consequences for rational and F-rational singularities of algebras with straightening laws.

This work is closely related to Objective 2 and Goal 4 of the DEFORMIALS project. It was written up in a preprint (see below), and submitted for publication at the Bulletin of the London Mathematical Society.

1.2. Versal deformations of toric singularities. With Klaus Altmann and Matej Filip we investigated deformations of affine toric singularities, viewed as germs around the closed orbit. Specifically, we aimed at describing the graded components of the versal base space. This is a generalization of [Alt97] and [AK13]. In the first two papers [AC19a, AC19b] we started by looking at “flat diagrams” of affine semigroups, and found a universal object in the category of flat extensions. This universal object is at the core of the whole project. It is defined for any rational polyhedron and generalises the tautological cone of Minkowski summands from [Alt97]. Inside this cone we define a lattice which encodes so-called “lattice friendly” Minkowski decompositions of the original polyhedron. These decompositions correspond to the components of the (graded piece of the) versal deformation of the toric singularity. The third paper [ACF19] deals with passing from the universal object to the equations of the graded components of the versal base space.

A key step in the proof of universality of our construction is given by associating a monomial ideal to a (a priori infinitely generated) semigroup. This aspect extends on the main theme of the INdAM project, which aims to connect deformation theory and monomial ideals. This work is also loosely related to Objectives 1 and 2 of DEFORMIALS, in the sense that it exploits all the combinatorial information in toric singularities to describe their deformations. Furthermore, the abstract flatness condition we proved for semigroup rings can be used to study deformations of semigroup rings of matroids.

This work has been written up in a long manuscript (65 pages), which we decided to split up for publication in three papers. We aim to submit the preprints for publication by the end of 2019 to mathematical journals of the highest level.

1.3. Deformations and the Macaulay inverse system. By Matlis duality, an Artinian quotient of the polynomial ring is Gorenstein if and only if we quotient by the ideal of a system of polynomial differential operators with constant coefficients having a unique solution. Elias and Rossi [ER17] very recently extended this characterization to Gorenstein quotients of arbitrary Krull dimension. With Maria Evelina Rossi and Laura Tozzo we are trying to construct Gorenstein ideals which contain a given Cohen-Macaulay ideal. While we dream of a method to do this in the most general case, our short term target is to do this for ideals of generic points in projective 3-space. This is the setting of one of the main open cases of the central conjecture in G-liaison theory. Eisenbud, Hartshorne and Schreyer [EHS15] proved the conjecture for $d$ generic points in $\mathbb{P}^3$ when $d \leq 33$ and $d = 37, 38$. Our strategy is different from theirs: we plan to solve the monomial Artinian case, then use a (Gröbner) deformation to reach the generic Artinian case, and then use the results of Elias and Rossi [ER17] to lift this to 1-dimensional homogeneous quotients of $C[x_0, x_1, x_2, x_3]$.

A key step in our approach above is to move from the monomial case to the generic case. In particular, for a pair of schemes $X \subset G$, with $G$ arithmetically Gorenstein, we want to find deformations of $G$ which are still Gorenstein and contain deformations of $X$. We plan to solve the corresponding problem for Artinian quotients first, and then lift it to zero-dimensional schemes. In the Artinian case, the particularities of deformations of monomial ideals, and those of hypersurfaces should play a key role. The latter correspond via the Macaulay inverse system to Artinian Gorenstein algebras. The technical aspects describing the Macaulay inverse system of Gorenstein algebras in higher dimension are surprisingly similar those which control obstructions to deformations.

This work is related to Objective 3 of DEFORMIALS, in the sense that it looks for connections of deformations with seemingly unrelated topics. Part of this work has been written up, but the project is not yet finalized.

2. Organization of Scientific and Popularisation Events. In September 2017 I was among the main organizers of two important events:

2. IMAGINARY exhibition at George Enescu Music Festival, Bucharest, 8-24 September 2017 
3. IMAGINARY exhibition at Festival della Scienza, Genova, 26 October-5 November 2017.

The first was one of the main events of 2017 in algebraic geometry world wide. It consisted of a one-week summer school with excellent lecturers: David Eisenbud (UC Berkeley), June Huh (Princeton), Peter
Scholze (U Bonn), Duco Van Straten (U Mainz). The second week was a conference with speakers of a similar level (including two from the University of Genova: Aldo Conca and Maria Evelina Rossi). Shreedevi Masuti and Matteo Varbaro from U Genova also participated in the event.

The second and third events were aimed at presenting mathematics, and algebraic geometry in particular, to a general public. The means chosen for this were artistic images and 3D prints of algebraic surfaces, and interactive touchscreen stations. The first exhibition took place in the foyer of one of the concert halls of the Enescu Festival in Bucharest. The second exhibition took place in the Loggia degli Abati of the Palazzo Ducale and had over 4000 visitors. Both exhibitions received a lot of press in the Romanian, Italian, and international media. INdAM was mentioned on all these occasions.

3. Conference and Seminar Talks.
11/2016 Deformations of Monomial Ideals, Algebra and Geometry Seminar, University of Genova (Italy)
04/2017 Castelnuovo-Mumford Regularity Vs. Virtual Cohomological Dimension
British Mathematics Colloquium - Algebra Workshop, Durham (UK)
06/2017 Linear syzygies, hyperbolic Coxeter groups, and regularity
Encounter in Topology ’n Algebra, Catania (Italy)
08/2017 Castelnuovo-Mumford Regularity vs. Virtual Cohomological Dimension
SIAM Conference on Applied Algebraic Geometry - GIT Atlanta (USA)
09/2017 Castelnuovo-Mumford Regularity vs. Virtual Cohomological Dimension
National School of Algebra, Bucharest (Romania)
11/2017 Linear Syzygies of Coxeter Groups
12/2017 Linear Syzygies and Hyperbolic Coxeter Groups
CMS Meeting - “Appl. of Comb. Top. in Comm. Alg.,” section, Waterloo (Canada)
12/2017 Commutative Algebra Seminar, University of Illinois at Chicago (USA)
12/2017 Algebraic Geometry/Commutative Algebra Seminar, Notre Dame University (USA)
01/2018 Humboldt Algebraic Geometry Seminar, Humboldt-Universität zu Berlin (Germany)

4. Publications and Preprints.
(1) Linear Syzygies, Flag Complexes, and Regularity (with T.Kahle, M.Varbaro)
(2) Gorenstein Liaison for Toric Ideals of Graphs (with E.Gorla)
(3) Linear Syzygies, Hyperbolic Coxeter Groups, and Regularity (with T.Kahle, M.Varbaro)
(4) On Smoothings of Combinatorial Manifolds (with E.De Negri, M. Varbaro)
(5) Polyhedra, Lattice Structures, and Extensions of Semigroups (with K.Altmann) (available on request)
(6) Universal Extensions of Pairs of Semigroups (with K.Altmann) (available upon request)
(7) Versality in Toric Geometry (with K.Altmann, M. Filip) (available upon request)

References