INDAM-DP-COFUND-2015 PROGRAMME

(first report)

Dear the president of INdAM fellowship,

I would like to submit my report on the research work performed respectively during the first year of the fellowship. My name is Nguyen Nhu Ngoc, I am a Ph.D student of mathematics department at Politecnico di Milano, Italy.

Affiliate: Department of Mathematics "Francesco Brioschi".

Ph.D course: Mathematics Models and Methods in Engineering.

My supervisor: Professor Silvia Lorenzani

First I would like to introduce my report covering the following sections:

1. **The title of my project:** Variational solutions of the linearized Boltzmann equation for thermal – driven gas flows in microchannels.

The micromachinery fabrication techniques have become more and more mature in the last few decades. In particular, the microelectromechanical system (MEMS) devices (inertial sensors, accelerometers, gyroscopes) developed rapidly and found many applications in microelectronics, medicine, biology, optics and other high technology fields. Micro- and nanodevices are often operated in gaseous environments (typically air), and thus their performances are affected by the gas around them.

Since the smallest characteristic length of MEMS is comparable with (or smaller than) the mean free path of the gas molecules, the traditional computational fluid dynamics methods, based on the Euler or the Navier-Stokes equations, fail in predicting the flows related to these devices. Therefore, an accurate analysis of such microfluidic systems requires the solution of the Boltzmann equation, which describes the evolution of the velocity distribution of the gas molecules in non-equilibrium statistical mechanics.

In spite of their apparently complex structure, the basic constituent of a real MEMS device is the microchannel, the region between two parallel plates that can reveal many specific features of the low speed internal flows in microdevices.

Therefore, an important aspect of the matter is to have an approximate closed form solution for gas flow rates in microchannels in order to use it in applications.

To develop an accurate formula directly from kinetic theory there is a particularly useful technique, the variational method proposed by C. Cercignani in 1969, which applies to the integrodifferential form of the Boltzmann equation and can be used for any linearized Boltzmann model. A practical advantage of the variational principle is that it allows to write down simple approximate equations to be used in practical design by suitable analytical manipulations.

In the current PhD project, the variational technique is used to compute temperature-driven (thermal creep) gas flows in microchannels in a wide range of Knudsen numbers (defined by the ratio of the molecular mean free path to a characteristic length of the system), by considering the true linearized Boltzmann collision operator and general models of boundary conditions (which describe the interaction of the gas molecules with the solid walls).

The aim of this work is to interpret recent experimental data on thermal creep flows, in a wide range of rarefaction regimes useful in many MEMS applications, which have revealed large discrepancies with the findings proposed in literature.

2. Status of research plan

Since it is difficult, in general, to manage the Boltzmann collision operator as such, we have first applied the variational technique to the Boltzmann equation based on a simplified kinetic model: the Bhatnagar, Gross and Krook (BGK) model.

We have assumed, at the moment, the simplest form of the gas-wall boundary conditions, that is the complete diffusion model according to which the gas molecules are reemitted by the channel plates with the Maxwellian distribution of the wall. A comparison between our variational results and the numerical outputs a vailable in literature is actually under way.

The next step will be to apply the variational technique to the Boltzmann equation based on the true linearized collision operator for hard-sphere molecules in order to obtain a better approximation of real-gas behavior.

Moreover, more general boundary conditions will be implemented. In particular, we will focus upon Maxwell's scattering kernel, according to which a fraction of molecules undergoes a specular reflection, while the remaining fraction is diffused with the Maxwellian distribution of the wall.

This aspect deserves a particular attention since walls with different physical structures are fully used in designing micromechanical systems.

3. List of educational activities carried out during the first PhD career

	Course title	Lecturer	Date	Grade	Place
1	Semilinear elliptic equations	Verzini Gianmaria, and Soave Nicola	08 Nov 2017 - 31 Jan 2018	A	Politecnico di Milano (Mathematics Department)
2	Principle of maximum and applications to elliptical and parabolic differential equation of the second order	Monticelli Dario Daniele, Punzo Fabio	01 Feb 2018 - 31 Mar 2018	A	Politecnico di Milano (Mathematics Department)

3	Semigroups of linear operators and applications to evolution equations	Conti Monica, Pata Vittorino	15 Mar 2018 - 30 May 2018	B	Politecnico di Milano (Mathematics Department)
4	Scientific communication in English (*)	Biscari Paolo, Sluckin Timothy Jan	16 Apr 2018 30 Apr 2018	C	Politecnico di Milano (Ph.D school)
5	Epistemology of scientific and technical research (*)	Folli Maria Grazia, Chiodo Simona, Campioli Andrea, Zanelli Alessandra	16 Feb 2018 02 Mar 2018	A	Politecnico di Milano (Ph.D school)
6	High order method for the approximate of partial differential equation	Quarteroni Alfio Maria, Dede' Luca	08 Jan 2018 - 23 Feb 2018	in processing	Politecnico di Milano (Mathematics Department)

(*) A set of courses aiming to train the PhD candidates in soft and transferable skills were propesed from the PhD School of Politecnico di Milano.

4. Research activities

I attended several activities related to research as below:

- **13/12/2017: (Seminar) Physic and Mathematic,** took place at Politecnico di Milano.
  - Title: Homoenergetic solution for the Boltzmann equation.
    Speaker: Alessia Nota, Università di Bonn
  - Title: Validity of the Boltzmann equation: how to incorporate long-range effects into a collisional dynamics
    Speaker: Chiara Saffirio, Università di Zurigo
- **21/03/2017: (Seminar) Physic and Mathematic,** took place at Politecnico di Milano.

  Title: Collisions and Chaos in Boltzmann-Grad limit.

  Speaker: Prof. Sergio Simonella, (ENS lione)
- **05/02/2018: (Seminar)** Took place at Politecnico di Milano.
  Title: On the weak maximum principle in unbounded domains.

  Speaker: Prof. Italo Capuzzo-Dolcetta, Sapienza Università di Roma, GNAMPA-Istituto Nazionale di Alta Matematica
- **28/05/2018: Lesson of Prof. Barry Simon,** took place at Politecnico di Milano.
  Title: Spectral theory, sum rules and large deviations.
- **22/06/2018:** (Seminar) Mathematical And Physical Seminar of Milan, organized by University of Milan-Bicocca.
  Title: Vaporizing and freezing the riemann zeta function

Speaker: Prof. Terence Tao, University of California, Los Angeles.

- **07/07-08/07/2018:** I took part in MSCA Satellite Event ahead of the EuroScience Open Forum (ESOF) 2018 in Toulouse, France. This is one of the Marie Skłodowska-Curie Actions.

- **28/11 – 01/12/2018**: I attended the conference in L'aquila. This workshop aims at providing an overview of different research lines on Quantum Mechanics at the crossroad between Mathematical Physics and Analysis. It was mainly intended for PhD students and young researchers. There were three short courses (4 hours each).

	Course title	Lecturer	Date	Place
1	Universal dynamics for the logarithmic Schrödinger equation	Rémi Carles (CNRS & Université de Rennes 1)	29/11-01/12/2018	GSSI, L'Aquila
2	Bosonic mean-field limits and the quantum de Finetti theorem	Nicolas Rougerie (CNRS & Université Grenoble-Alpes)	28-30 /11/2018	GSSI, L'Aquila
3	Kinetic Equations describing Wave Turbulence	Juan J. L. Velázquez (University of Bonn)	28-30 /11/2018	GSSI, L'Aquila

On 21 October 2018, the Faculty Board of the mathematics department of Politecnico di Milano had an annual evaluation in order to addmit to Ph.D students. I have gotten grade **A** for the first year with the approval of my advisor. The website of the kinetic and field theory of Politecnico di Milano is https://www.kinetic.mate.polimi.it/people.html, which also mentions about my field what I am working on.

Finally, thanks for your support. My educational pursuits would not be possible without great support from scholarship sponsors like your organization.