

# INDAM-DP-COFUND-2015 FIRST RESEARCH WORK REPORT

KAMAL DIKI

Dear President of INdAM,

I am glad to inform you that the board of professors of the host PhD Programme admitted me at second year with passing grade A. This report summarizes the main research work progress and research activities carried out by the fellow at the host institution *Politecnico di Milano* under the project *INdAM Doctoral Programme in Mathematics and/or Applications Cofunded by Marie Skłodowska-Curie Actions*, grant number : 713485. The report includes the status of the research works performed from the start of the fellowship, a complete list of attended PhD courses, the main participations in scientific events including seminars, conferences, etc. It also contains some new research works that are still in progress.

## 1. GENERAL INFORMATION

- **Name of the fellow** : *Diki Kamal*
- **University** : *Politecnico di Milano*
- **Doctoral Programme** : *Mathematical Models and Methods in Engineering*
- **Coordinator** : *Prof. Sabadini Irene*
- **Supervisor** : *Prof. Sabadini Irene*
- **Tutor** : *Prof. Marchini Elsa*
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## 2. RESEARCH PROJECT PROGRESS

**Research project title** : *Segal-Bargmann transforms in slice and Clifford analysis*

**Status of research works** : In this research project, we study the subject of Segal-Bargmann-Fock spaces and related topics in the setting of complex, quaternions and Clifford analysis. It turns out that these mathematical models are very important for possible applications in mathematical physics, especially in quantum mechanics and also in signal processing.

Starting our research studies in mi-November 2017, under the supervision of *Prof. Sabadini* from *Politecnico di Milano*, we submitted three papers for publication in this field of study. Indeed, the first one is titled : « *The Cholewinski-Fock space in the slice hyperholomorphic setting* ». This paper studies an extension of the classical slice hyperholomorphic Fock space introduced in 2014 by Alpay, Colombo, Sabadini and Salomon. This extension is possible by considering on the space of slice entire functions a specific weight involving a modified Bessel function of the third kind, namely the Macdonald function. We give a complete description of this quaternionic Hilbert space. Then, its reproducing kernel is obtained making use of the slice hyperholomorphic extension of the classical complex Dunkl kernel. It was also possible to construct an associated unitary integral transform, and study some specific quaternionic operators on the slice hyperholomorphic

Cholewinski-Fock space. This construction follows an approach by Cholewinski in 1984.

The second paper is a joint work with my supervisor *Prof. Sabadini* and *Prof. Alpay* from *Chapman University*. The paper is titled : « *On slice polyanalytic functions of a quaternionic variable* ». In this research work, we propose a new definition extending to higher order the theory of slice hyperholomorphic functions on the quaternions originally introduced by *Gentili* and *Struppa* in 2007. We study some basic properties of these functions and prove in this context the counterparts of the following results : Splitting Lemma, Identity Principle, Representation Formula, Extension Lemma, Refined Splitting Lemma and present some of their consequences. Then, we consider the Fock and Bergman spaces in this new setting and compute explicit expressions of their reproducing kernels. In this same research direction, we would like to understand if a Segal-Bargmann transform on the quaternionic slice polyanalytic Fock space exists. In this case, it should extend some results obtained by *Abreu* in 2010 on the complex polyanalytic Segal-Bargmann transforms. This may require to study a quaternionic analogue of the well-known Gabor transform.

The third paper is a joint work with my supervisor *Prof. Sabadini* and *Prof. Gal* from the *University of Oradea*. The paper is titled: « *Polynomial approximation in slice regular Fock spaces* ». The Banach Fock spaces of slice hyperholomorphic functions on the quaternions are introduced, both of the first and of the second kind. It is shown that in the two theories it is always possible to approximate any function of these quaternionic Fock spaces by a sequence of quaternionic polynomials. This paper extends some classical results contained in the book of *Zhu* titled *Analysis on Fock spaces*.

At the moment, in collaboration also with *Prof. Krausshar* from the *University of Erfurt* we are studying some slice Bergman kernels and associated Bergman-Fueter transforms both on the quaternionic half space and half ball. An extension to the Clifford setting on these domains would be possible also thanks to the so called Bergman-Sce transform. In this same spirit, we focus our interest on a specific integral transform of Bargmann-Fock type in the monogenic setting on the quaternions. These constructions are based on the well-known Fueter mapping theorem. In particular, we show that it is possible to relate the normalized Hermite functions of the standard Hilbert space on the real line into a specific Appell system of quaternionic monogenic polynomials. As a consequence, we find some new quaternionic reproducing kernel Hilbert spaces, integral representations and generating functions related to the well-known Appell system of monogenic polynomials introduced in the last years by *Malonek* and his collaborators.

### 3. COURSES

During the academic year 2017-2018, I attended two PhD School courses, three courses from the department of mathematics at *Politecnico di Milano* and one summer school course at the *University of Bologna*. This allowed me to earn 25 credits and pass the first year, grade A. Here is the list of attended courses

#### 3.1. PhD School Courses :

- (1) *Epistemology of Scientific and Technical Research (passed, grade A)*. This course took place at *Politecnico di Milano* from 16 February 2018 to 02 March 2018. This course was organised by *Prof. Chiodo Simona* from *Politecnico di Milano*.

- (2) *Scientific Communication in English (passed, grade B)*. This course took place at *Politecnico di Milano* from 16 April 2018 to 04 May 2018. The lecturer of this course is *Prof. Tim Sluckin*, from the *University of Southampton*.

### 3.2. PhD Programme Courses :

- (1) *Semi-linear Elliptic Equations (passed, grade A)*. This course took place at the mathematical department of *Politecnico di Milano* from 08 November 2017 to 31 January 2018. The lecturers of this course are *Prof. Gianmari Verzini* and *Prof. Soave Nicola* from *Politecnico di Milano*.
- (2) *Maximum Principles and Applications to Second Order Elliptic and Parabolic Partial Differential Equations (passed, grade A)*. This course took place at the mathematical department of *Politecnico di Milano* from 01 February 2018 to 31 March 2018. The lecturers of this course are *Prof. Monticelli Dario Daniele* and *Prof. Punzo Fabio* from *Politecnico di Milano*.
- (3) *Semi-groups of Linear Operators and Applications to Evolution Equations (passed, grade A)*. This course took place at the mathematical department of *Politecnico di Milano* from 15 March 2018 to 30 May 2018. The lecturers of this course are *Prof. Conti Monica* and *Prof. Pata Vittorino* from *Politecnico di Milano*.

### 3.3. Summer School Courses :

- (1) ACOTCA 2018: Advanced Courses in Operator Theory and Complex Analysis, *University of Bologna*, June 11-14, 2018.

## 4. SCIENTIFIC ACTIVITIES

4.1. **Seminars** : I was able to attend several mathematical seminars and lectures that took place at *Politecnico di Milano*, *University of Bicocca* and *University of Milano*. Here is a list of some selected seminars

- (1) One-Slice Preserving Functions of a Quaternionic Variable, by *Prof. Chiara de Fabritiis* from *Università Politecnica delle Marche*, 15 March 2018 at *Dipartimento di Matematica, Politecnico di Milano*.
- (2) Spectral Theory, Sum Rules and Large Deviations, by *Prof. Barry Simon* from *California Institute of Technology*, 28 May 2018 at *Dipartimento di Matematica, University of Milano*.
- (3) Tales of Our Forefathers, by *Prof. Barry Simon* from *California Institute of Technology*, 29 May 2018, at *Dipartimento di Matematica, Politecnico di Milano*.
- (4) On the harmonicity of slice regular functions, by *Prof. Cinzia Bisi* from *Università di Ferrara*, 31 May 2018 at *Dipartimento di Matematica, Politecnico di Milano*.
- (5) The essential norm estimates of Hankel and the  $\bar{\partial}$ -Neumann operators, by *Prof. Zeljko Cuckovic* from *University of Toledo*, 01 June 2018 at *Dipartimento di Matematica, University of Milano*.
- (6) Vaporizing and Freezing the Riemann Zeta Function, by *Prof. Terence Tao* from *University of California, Los Angeles*, 22 June 2018 at *University of Bicocca*.

4.2. **Conferences** : I attended and gave a talk in two international conferences in Germany and US.

- (1) International Conference on Hypercomplex Analysis and Applications, *Erfurt University*, Germany, September 24-26, 2018. Contributed talk title : *On slice polyanalytic functions of a quaternionic variable*.
- (2) Advances in Operator Theory with Applications to Mathematical Physics, *Chapman University*, Orange CA, US, November 12-16, 2018. Contributed talk title : *On slice polyanalytic functions of a quaternionic variable*.

4.3. **Special events :** I participated in a special event for Marie-Sklodowska Curie fellows that took place in France.

- (1) MSCA Satellite Event ESOF 2018, Toulouse, France, July 7-8, 2018.

## 5. PUBLICATIONS

### 5.1. Submitted manuscripts :

- (1) On slice polyanalytic functions of a quaternionic variable. ( Joint work with *Prof. Alpay, D* and *Prof. Sabadini, I.*)
- (2) Polynomial approximation in slice regular Fock spaces. ( Joint work with *Prof. Gal, S* and *Prof. Sabadini, I.*)
- (3) The Cholewinski-Fock space in the slice hyperholomorphic setting.

### 5.2. Research works in progress :

- (1) On the Bargmann-Fock-Fueter and Bergman-Fueter integral transforms.
- (2) Generalized Fock spaces on quaternions.