Report of the academic activity during the first year of the PhD programme

PhD Student: Chalmoukis Nikolaos
Supervisor: Prof. Nicola Arcozzi
Tutor: Prof. Nicola Arcozzi
Circle: 33°

1. Courses and seminars

As part of the first year of the PhD programme I followed two courses of total duration 30h each. In particular,

- *Hodge Theory*
  Professor: Luca Migliorini

- *Partial Differential equations*
  Professor: Maria Manfredini

Furthermore I participated in the Summer School "Mathematics in Image Science" at the University of Bologna. Also, I followed the series of seminars *Topics in Mathematics I* and other seminars organized by the department, in particular the ones organized by the Complex Analysis Lab, the group of complex analysis of the U. of Bologna, of which I am a member.

3. Conferences and Summer Schools

- *Workshop su varietá reali e complesse: Geometria, topologia e analisi armonica*
  Place: Pisa, Scuola Normale Superiore
  Period : February 1 2018 - February 3 2018
  The conference was the annual meeting inside the PRIN project.

- *Analysis Days in Piemonte*
  Place: Piemonte
  Period: May 7 2018 - May 11 2018
  A workshop organized by the Chebyshev Laboratory in collaboration with the department of mathematics of the University of Bologna. I gave a talk entitled "Generalized Integration operators on Hardy spaces"

- *Workshop on complex analysis and operator theory*
  Place: Blanes, Spain
  Period: May 26 - May 30 2018
  A summer school and workshop organized by the node of the Universitat Autónoma de Barcelona. I took part to the course "Quasiconformal mappings and the Beurling transform" given by Professor Maria Jose Gonzalez, and I gave a talk entitled "Unions of onto interpolating sequences for the Dirichlet space".

- *Advanced courses in operator theory and complex analysis*
  Place: University of Bologna
  Period: June 11 - June 14 2018
  A conference co-organized be the departments of mathematics of the U. of Bologna, U. Paris-Est Marne-la-Vallee, France and ICMat e U. Complutense de Madrid, Spain. I participated as a local organizer and I gave a talk entitled "Unions of onto interpolating sequences for the Dirichlet space".

- *New developments in complex analysis and function theory*
  Place: University of Crete, Greece
  Period: July 2 - June 6 2018
  A conference concentrated on the topics of Complex Dynamics Geometric Function Theory and Operator Theory Interactions between Complex Analysis and Probability and Potential Theory and Approximation.

4. Talks

Apart from the talks mentioned in the previous paragraph, I also gave the following ones.

- *Generalized Integration operators on Hardy spaces*
  Place: U. of Bologna
  Date: November 10 2017
  As part of the activities of the Complex Analysis Lab

- *Onto iterpolating sequences for the Dirichlet space*
  Place: Aristotle U. of Thessaloniki
  Date: September 27 2018

4. Research activity

My general research area is Banach spaces of analytic functions and operator theory on them. In particular one of the main objects we interested in studying, my self personally and in general our research group, is the Dirichlet space $\mathcal{D}$ of analytic functions, and some weighted versions of it. The theory of the Dirichlet space is not as well developed as the corresponding theory of Hardy or Bergman spaces for example and there are major open problems concerning this class of functions, to mention for example the the Shapiro-Shields conjecture on the invariant subspaces of $\mathcal{D}$.

In this first period, I was occupied with the study of the onto interpolating sequences for the Dirichlet space. Interpolation problems where introduced first in relation with the algebra of bounded analytic functions $H^\infty(\mathbb{D})$ by L. Carleson and then generalized in many different directions. In the Dirichlet space, there exist two possible ways one could generalize the question of interpolation, and they lead to two different type of interpolating sequences, one called Universal Interpolating Sequences (UI) and the other one Onto Interpolating Sequences (OI). The (UI) sequences have been characterized in previous works of C. Bishop, D. Marshall, C. Sundberg [1] and [4], where they also gave some sufficient conditions for a sequence in order to be (OI). But the general problem of characterizing (OI) sequences is still open.

My contribution was to characterize (OI) sequences under some additional assumption concerning the finiteness of a measure . The characterization is given in terms of the logarithmic capacity of some sets or equivalent the capacity of some condensers constructed appropriately. This work is now in a preprint version [3]. My results are based on a combination of techniques which can be traced in works of C. Bishop, B. Böe, D. Marshall and C. Sundberg ([2], [1], [4]).

In this direction, the, partial at least, understanding of (OI) sequences for $\mathcal{D}$ has open up a multitude of possible research directions, regarding alternative characterizations of (OI) sequences, sampling sequences or Carleson measures, just to mention some.

4. Preprints and Submitted papers

- N. Chalmoukis, Onto Interpolation for the Dirichlet space and for $W^{1,2}(\mathbb{D})$, 2018arXiv180708193C. (Submitted to "Advances in Mathematics".)

- N. Chalmoukis and M. Levi, Some Remarks on the Dirichlet Problem on Infinite Trees, 2018arXiv181102263C. (Submitted to "Concrete Operators" as a contribution to the ACOTCA 2018 conference proceeding volume.)

References

[1] C. J. Bishop, *Interpolating sequences for the dirichlet space and its multipliers*, 1994.

[2] B. Böe, *Interpolating sequences for besov spaces*, Journal of Functional Analysis, 192 (2002), pp. 319 – 341.

[3] N. Chalmoukis, *Onto Interpolation for the Dirichlet Space and for $W^{1,2}(\mathbb{D})$*, ArXiv e-prints, (2018).

[4] E. D. Marshall and C. Sundberg, *Interpolating sequences for the multipliers of the dirichlet space*, https://sites.math.washington.edu/ marshall/preprints/interp.pdf, (1994).