FINAL REPORT OF MARTA STRANI

This is my research activity of the last year, during the second year of the INdAM Fellowship cofunded by Marie Curie Actions: it has been carried out both in Paris (University of Paris Diderot) and during the return phase in Roma, Università la Sapienza.

Concerning the first part, together with B. Texier we have focused on the influence of a regularizing term in the instabilities of hyperbolic systems: precisely, we considered systems that, because of their structure, present both hyperbolic and elliptic zones so that the problem is ill-posed, and we investigate how the introduction of a regularizing term influences these instabilities.

In particular, we studied a Shröedinger regularization of the Euler system in Lagrangian coordinates

$$\begin{cases} \partial_t v + \partial_x u = 0, \\ \partial_t u + \partial_x p(v) = \varepsilon i \partial_x^2 v. \end{cases} \tag{0.1}$$

This regularization have the effect of not modifying the conservation law that holds at the level $\varepsilon = 0$. The downside, of course, is that we lose the real character of u and v.
The underlying instability is given by a Van der Waals equations of state, corresponding to a pressure law $v \to p(v)$ such that $p'(v) < 0$ for some v. For the inviscid problem, it has been shown that the eigenvalues of the first-order symbol are not real, so that the first-order system is strongly ill-posed.

The question is whether the dispersive term will allow us to prove local and/or global well posedness. In [6], we proved a first local-in-time existence result for solutions arising from initial data that are highly oscillating: the proof is based on a normal form reduction and on the use of Strichartz estimates.

Concerning my return phase in Roma, I continued working on the phenomenon of metastability: this behavior appears when the time dependent solutions of an evolutive PDE exhibit a first transient phase where they are close to some non-stationary state before converging, in an (exponentially) long time, to their asymptotic limits.

At the moment I'm currently working together with M. Garrione (Università di Milano Bicocca) and Raffaele Folino (Univeristà degli studi dell'Aquila) on a paper where we address this problem for convection diffusion equation with a nonlinear diffusion of mean-curvature type. In a previous paper ([4]) we showed existence of solutions of traveling wave type for the problem in the whole real line; what is more, we showed that the speed rate of these solutions is proportional to the viscosity parameter ε, this being the reason why we expect that a metastable behavior appears if we restrict our analysis to a bounded interval of the real line.
So far, in [3] we have proved existence and stability of a stationary solution under some smallness assumption on the boundary conditions; next step will be to give an estimate for the slow speed of convergence of the time dependent solution towards this stable configuration.

Other results that I have obtained during this period are [1, 2, 5]. In particular, in [5] we extend the results contained in [4] to a diffusion of (p, q)-Laplacian type; in [2], we describe a general procedure to prove metastability in convection-diffusion equations.

References

[1] De Luca, L.; Goldman M.; Strani M.; *Stability of the kink for the Cahn-Hilliard equation for* $1 \leq d \leq 5$, in preparation.

[2] Folino, R.; Lattanzio, C.; Mascia, C.; Strani, M.; *Metastability for nonlinear convection-diffusion equations*, NoDEA 24 no. 4 (2017).

[3] Folino, R.; Garrione, M.; Strani, M.; *Slow convergence and stability of the steady states in viscous conservation laws with mean curvature type diffusion*, in preparation.

[4] Garrione, M.; Strani, M.; *Heteroclinic traveling fronts for quasilinear Fisher-Burgers equations* , submitted.

[5] Garrione, M.; Strani, M.; *Monotone wave fronts for* (p, q)*-Laplacian driven reaction-diffusion equations*, submitted.

[6] Strani, M.; Texier, B.; *Dispersive regularization of a Van der Waals gas*, in preparation.