

visKO - Visualization of Knotted Objects
Final Report
INdAM-COFUND-2012
1 January 2015 - 31 December 2016
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The main purpose of the project *visKO* was to realize a movie in computer graphics explaining some topics in 4-dimensional topology at a level suitable for popularization and dissemination of research results. The movie “Knots in 4D” can be viewed at <https://www.youtube.com/watch?v=Lx85kIGjIoQ&list=PlyxHTRWELFBr9TP8jx400bPGP-ECsBufz>. It is divided into three parts, each one lasts about 20 minutes. The titles are: Knots, Knotted surfaces, and Ribbon surfaces. The first and second parts are strongly linked, they describe the corresponding concepts in dimensions 3 and 4: isotopies, broken arc/surface diagrams, Reidemeister/Roseman moves to describe isotopies, the problem of classification, a simple invariant: the three-colorability. The third part presents two definitions of ribbon knotted surfaces, a diagrammatic description, and Satoh’s Tube map. It ends with an open question on the injectivity of the map.

The short movie “Spinning in various dimensions” introduces the class of spun knotted surfaces without using words. It was submitted to the MathLapse competition at the conference Imaginary in Berlin in July 2016 and selected for the final phase. It can be viewed at <https://www.youtube.com/watch?v=-TuXJswaFAE>.

Publications

- *Using braids to introduce groups: From an informal to a formal approach.* Proceedings of the 13th International Congress on Mathematical Education (2016), Springer.
- *Matematica e visualizzazione: un’esperienza didattica*, in F. Broglia, M. Dedò and I. Tamanini, editors, *Vedere la matematica alla maniera di Mimmo Luminati*. Edizioni ETS, Pisa, 2015.
- *Proposte di laboratorio sui fondamenti della matematica: l’infinito e i sistemi assiomatici*, submitted.

Talks

- 29 July 2016. Workshop *Braids and Groups*, 13th International Congress on Mathematics Education, Hamburg, Germany.
- 20–23 July 2016. Leader of the 4-days workshop *Spherical geometry in a science museum*, Imaginary conference, Berlin, Germany.

- 30 April 2016. *A movie on knotted surfaces in 4D*. Knots in The Triangle/Knots in Washington XLII, Raleigh, NC.
- 13 March 2016. *Visualization of knotted surfaces in 4D*. Advances in Quantum and Low-Dimensional Topology 2016, Iowa City, IA.
- 26 May 2015. *A classification of welded knots up to 4 crossings*, INdAM Meeting Chromatic and colored structures in geometry and statistical physics, Cortona, Italy.

Schools and conferences attended

- ICME13, 13th International Congress on Mathematics Education, Hamburg, Germany, July 24–31, 2016.
- Imaginary conference, Berlin, Germany, July 20–23, 2016.
- Illustrating Mathematics Workshop, ICERM, Brown University, Providence, RI, June 27 – July 1, 2016.
- Topology Festival, Cornell University, Ithaca NY, May 13–15, 2016.
- New Developments in TQFT, QGM Aarhus (Denmark), July 27–31, 2015.
- Swiss Knots 2015, Geneva (Switzerland), June 1–3, 2015.
- INdAM Meeting “Chromatic and colored structures in geometry and statistical physics”, Cortona (Italy), May 24–30, 2015.

Selected education and popularization activities

- Since November 2016, curator of the website <http://www.xlatangente.it/>, managed by the the research center *matematita*. Description in Italian of some of the MathLapse, short movies presented at the Imaginary conference in Berlin in 2016, <http://www.xlatangente.it/page.php?id=1630>.
- Two short movies selected for the final session in the MathLapse competition <https://imaginary.org/films/mathlapse> at the Imaginary conference, Berlin.
- 6-10 June 2016, Instructor at the UGA MathCamp for high school students, Athens, Georgia (USA). <http://tossor.github.io/mathcamp/#node01500101mathcamp2016>.
- Winter term 2016, mentor for a gifted high school student, outreach program of the University of Toronto.
- 18 November 2015. Talk “Visualization of 4-dimensional knots” for undergraduates in mathematics at the University of Toronto.

- 2015 and 2016. Various workshops (braids, axiomatic systems, symmetries) for middle and high school students in the outreach program of the University of Toronto.
- “Anelli volanti, lombrichi e cristalli”, interview with Dror Bar-Natan, published in XlaTangente n.3, March 2015, pages 6–9.