



# The final report

## INdAM COFUND fellowship

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### Summary of the work

Our main goal is to obtain conservativity of Bar induction over dependently typed theory like Minimalist Foundation.

We (the fellow) could not achieve our major goal, but made some progress. We show the derived rule of bar induction for the simply typed theory (Conference talks 1, 3). We consider this result to be a good step toward our main goal. We are currently trying to extend this result to dependently typed theories. We also plan to generalise this result so that we can achieve several types of derived rules in a single framework.

We are still working with the researchers of the hosting institution towards our main goal. Our next step is to achieve our goal for the finite type system, which we hope to reach during next year.

On the other hand, we made a good progress in understanding the logical property of Bar induction. In Publication 2 and Submitted work 2, we characterised Bar induction which captures the difference between the classical notion of pointwise continuity and constructively fundamental notion of continuity called Brouwer continuity. A part of this is a joint work with Makoto Fujiwara (Waseda University). Subsequently (in Publication 3), we showed that Brouwer continuity is equivalent to the continuity notion in formal topology. We also applied the insight from these works to obtain a natural characterisation of  $\Pi_1^0$  fan theorem (Publication 1). Moreover, together with Giovanni Sambin of the hosting institution, we characterised bar induction in terms of continuity principle in the setting of constructively topology (Submitted work 4).

### Other activities

Besides the work reported above, we worked on several topics on pointfree topology. These are the works that we started just before the fellowship began and continued during the fellowship.

First, in a joint work with Matthew de Brecht (Kyoto University), we showed several interesting properties of powerspaces of quasi-Polish spaces, one of the promising notions in descriptive set theory (Submitted work 1, Conference talks 5). Second, we showed that the notion of located subsets corresponds to that of Lawson topology, thus connecting one of the central notions in constructive topology and domain theory (Submitted work 3, Conference talks 3, 6, 7). Third, together with Francesco Ciraulo and Samuele Maschio of the hosting

institution, we gave a characterisation of the constructive notion of topology called *basic topology* in term of categorical logic (Conference talk 2).

Finally, we organised several conferences:

- *XXVI incontro dell'Associazione Italiana di Logica e sue Applicazioni*. Padova Italy, 25–28 September, 2017 (with Cinzia Bonotto, Francesco Ciraulo, Maria Emilia Maietti, Samuele Maschio, and Fabio Pasquali)
- *Second Workshop on Mathematical Logic and its Application*. Kanazawa, Japan, 5–9 March 2018 (with Ryota Akiyoshi, Makoto Fujiwara, Takako Nemoto, and Keita Yokoyama)

## Publications

1. T. Kawai, *A continuity principle equivalent to the monotone  $\Pi_1^0$  Fan theorem*, Accepted for publication in Archive for Mathematical Logic.
2. T. Kawai, *Principles of bar induction and continuity on Baire space*, Accepted for publication in Journal of Logic & Analysis.
3. T. Kawai, *Formally continuous functions on Baire space*, Mathematical Logic Quarterly, 64(3), 192–200 (2018)
4. T. Kawai, *Localic completion of uniform spaces*, Logical Methods in Computer Science, Volume 13, Issue 3 (September 13, 2017)
5. T. Kawai, *A point-free characterisation of Bishop locally compact metric spaces*, Journal of Logic & Analysis 9:c2, 1–41 (2017)
6. T. Kawai, *Point free characterisation of Bishop compact metric spaces*, Journal of Logic & Analysis 9:5, 1–30 (2017)

## Submitted works

1. M. de Brecht and T. Kawai, *On the commutativity of the powerspace constructions*.
2. M. Fujiwara and T. Kawai, *A Logical Characterisation of the Continuous Bar Induction*.
3. T. Kawai, *Geometric theories of patch and Lawson topologies*.
4. T. Kawai and G. Sambin, *The principle of point-free continuity*.

## Conference talks

1. T. Kawai. *On the domain of definable functionals of Gödel's system T*, Workshop: Constructive Mathematics, Bonn, Germany (August 2018)
2. F. Ciraulo and T. Kawai and S. Maschio. *Some doctrines on suplattices and basic pairs*. Logic Colloquium 2018, Udine, Italy (This work was presented by S. Maschio).

3. T. Kawai. *On the existence of general bar recursors*. Proof, Computation, Complexity 2018, Bonn, Germany (July 2018)
4. T. Kawai. *Applications of entailments: de Groot duality*. Workshop DOMAINS, Oxford, UK (July 2018)
5. T. Kawai and Matthew de Brecht. *Duality of upper and lower power-locales on locally compact locales* Continuity, Computability, Constructivity (CCC 2017) Nancy, France (June 2017)
6. T. Kawai. *Lawson Topology as the Space of Located Subsets*, Topology, Algebra, and Categories in Logic 2017 Prague, Czech republic (June 2017)
7. T. Kawai. *The space of located subsets*, Second CORE meeting, LMU München, Germany. (January 2017)