ABSTRACTS

INdAM Workshop *Complex function theory, its generalizations and applications*

Roma, September 12th-16th, 2016

Nicola Arcozzi (Università di Bologna)
The Dirichlet Space on the bi-disc

We present some preliminary findings on properties of the Dirichlet space on the bi-disc, which is defined as the tensor product of two copies of the classical Dirichlet space on the unit disc. Work in collaboration with Pavel Mozolyako, Karl-Mikael Perfeckt, Giulia Sarfatti.

Swanhild Bernstein (TUB Freiberg)
Generalized Riesz-Hilbert transforms and their applications

The Hilbert transform $Hf(x) = \frac{1}{\pi}\, p.v. \int_{-\infty}^{\infty} \frac{f(y)}{x-y} dy$ is a very important transform in complex analysis. It allows to define boundary values of analytic functions and builds up the analytic signal $f + iHf$ and it is the role model of a singular integral operator. The generalization into higher dimensions is given by the Riesz transforms

$$R_j f(\underline{x}) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\pi^{(n+1)/2}} \int_{\mathbb{R}^n} \frac{y_j}{|\underline{y}|^{(n+1)/2}} u(\underline{x} - \underline{y})\, d\underline{y}, \quad \underline{x},\, \underline{y} \in \mathbb{R}^n.$$

Using quaternions of Clifford algebras the Riesz transforms can be combined to an operator which we will call Riesz-Hilbert operator $H = \sum_{j=1}^{n} e_j R_j f$. The Riesz-Hilbert operator describes boundary values of monogenic functions, can be used to build the monogenic signal and is a singular integral operator. Furthermore, the Riesz-Hilbert transform connects the Dirac operator with the fractional Laplacian of order $\frac{1}{2}$ because $D = |\Delta|^{1/2} H = |D| H$.

Driven by an application in optics we introduce fractional Riesz-Hilbert transforms. The Riesz-Hilbert transform commutes with rotations, but for special applications it would be necessary that the transform commutes with shearings. It can be proven that a modified (linearized) Riesz transform H_L has this properties and we can use this Riesz-Hilbert transform H to define quasi-monogenic functions which have analog properties like monogenic functions.

Fabrizio Colombo (Politecnico di Milano)
Quaternionic spectral theory

In this talk we give an overview of the quaternionic spectral theory based on the notion of S-spectrum. We present the state of the art of the quaternionic version of the various functional calculi associated with slice hyperholomorphic functions. Moreover we discuss the spectral theorem for quaternionic (unbounded) normal operators using the notion of S-spectrum. The proof consists of first establishing a spectral theorem for quaternionic bounded normal operators and then using a transformation which maps a quaternionic unbounded normal operator to a quaternionic bounded normal operator. With the spectral theorem we complete the foundation of spectral analysis of quaternionic operators. An important motivation for studying the spectral theorem for quaternionic unbounded normal operators is given by the subclass of unbounded anti-self adjoint quaternionic operators which plays a crucial role in the quaternionic quantum mechanics.

Manuel D. Contreras (Universidad de Sevilla)
Integral operators mapping into the space of bounded analytic functions

Let g be an analytic function in the unit disk $\mathbb{D}$. We consider the integral operator T_g defined by

$$T_g(f)(z) = \int_0^z f(\xi)g'(\xi)\,d\xi$$

for all analytic functions $f : \mathbb{D} \to \mathbb{C}$. In 1977, Ch. Pommerenke got that T_g is bounded on the Hardy space H^2 if and only if g belongs to BMOA (the space of analytic functions of bounded mean oscillation). Since that year, a number of authors has studied several properties like boundedness, compactness, ... of $T_g : X \to Y$ between two Banach spaces of analytic functions X and Y.

In this talk, we present some new results about $T_g : X \to H^\infty$ where H^∞ is the space of bounded analytic functions in the unit disk. We will discuss boundedness, compactness and weak compactness when X is either the Bloch space, BMOA, a Hardy space H^p, $1 \leq p \leq +\infty$, or a Bergman space $\mathcal{A}^p_\alpha$, $1 \leq p < +\infty$ and $\alpha > -1$.

This is a joint work with J.A. Peláez, Ch. Pommerenke, and J. Rättyä.

Hendrik De Bie (Universiteit Gent)
Uni- and multivariate discrete orthogonal polynomials using Dirac operators

Discrete orthogonal polynomials have been classified in the so-called Askey scheme of orthogonal polynomials. Since many years their continuous counterparts have found ample applications in Clifford analysis.

In the present talk I will show that also discrete polynomials appear naturally and unavoidably, when studying certain Dirac models, as expansion coefficients between different orthonormal bases for spaces of spherical monogenics.

For the univariate case we will use a generalized Dirac operator in 3D, while for the multivariate case we have to resort to nD. Along the way, we will construct a new symmetry algebra that can be interpreted as a higher rank version of the Bannai-Ito algebra.

Franc Forstnerič (Univerza v Ljubljani)
New complex analytic methods in the study of non-orientable minimal surfaces in $\mathbb{R}^n$

I will explain how to adapt the complex analytic methods originating in modern Oka theory to the study of non-orientable conformal minimal surfaces in $\mathbb{R}^n$ for any $n \geq 3$. In particular, we obtain Runge-Mergelyan type approximation theorems, general position results, the construction of proper non-orientable conformal minimal surfaces, and of complete minimal surfaces bounded by non-rectifiable Jordan curves.

Riccardo Ghiloni (Università di Trento)
Algebraic structure and zero sets of slice functions

The goal of this talk is to present some fundamental algebraic properties of slice functions and slice regular functions over a real alternative *-algebra A. These function theories have been introduced in 2011 as a higher-dimensional generalization of the classical theories of functions and holomorphic functions of a complex variable, of the theory of slice regular quaternionic functions launched by Gentili and Struppa in 2006 and of the theory of slice monogenic functions constructed by Colombo, Sabadini and Struppa since 2009. The set of slice functions over A, which includes all polynomials over A, forms an alternative *-algebra itself when endowed with appropriate operations. We study this algebraic structure. In particular we investigate the existence of multiplicative inverses by means of a detailed study of the zero sets of slice functions and slice regular functions, which are of independent interest. These recent results have been obtained jointly with Alessandro Perotti (Università di Trento) and Caterina Stoppato (Università di Firenze).

Pamela Gorkin (Bucknell University, Lewisburg)
The numerical range, Blaschke products and compressions of the shift operator

The numerical range of an $n \times n$ matrix A is the set

$$\{\langle Ax, x\rangle : x \in \mathbb{C}^n, \|x\| = 1\}.$$

In this talk, we look at the numerical range of certain matrices that have particularly nice geometric properties. These matrices represent compressions of the shift operator acting on finite dimensional model spaces, $K_B = H^2 \ominus BH^2$, where B is a finite Blaschke product and H^2 is the Hardy space. Using the complex analysis point of view, we obtain new information about the numerical range of these operators. Conversely, the numerical range allows us to obtain new results about finite Blaschke products.

Klaus Gürlebeck (Bauhaus-Universität Weimar)
On interpolation with monogenic polynomials - theory and applications

Joint work with Dmitrii Legatiuk (Bauhaus-Universität Weimar).

We study the general problem of interpolation of monogenic functions by monogenic polynomials or special systems of more general basis functions. The best result will be stated in terms of optimal interpolating functions. Some constructive aspects of this approach will be discussed, supported by concrete examples. A special point is the connection of interpolation and best approximation which is related also to the properties of Appell systems of homogeneous monogenic polynomials. In the second part of the talk we focus the attention to special constructions, based on Pseudo Complex Powers. The main problem is to find interpolation polynomials which converge to general monogenic functions under weak conditions for the interpolation nodes.

Matvei Libine (Indiana University, Bloomington)
Anti de Sitter deformation of quaternionic analysis and the second order pole

I will talk about the quaternionic analogues of Cauchy's formula for the second order pole. These quaternionic analogues are closely related to regularization of infinities of vacuum polarization diagrams in four-dimensional quantum field theory.

Then I will describe a one-parameter deformation of quaternionic analysis. This deformation of quaternions preserves conformal invariance and has a geometric realization as anti de Sitter space sitting inside the five-dimensional Euclidean space. Many results of classical quaternionic analysis - including the Cauchy-Fueter formula - extend to this new setting.

This is a joint work with Igor Frenkel from Yale University.

Marco Peloso (Università di Milano)
On the Diederich-Fornaess index of a domain

Let Ω be a smoothly bounded pseudoconvex domain in $\mathbb{C}^n$. The Diederich-Fornaess index of Ω is defined as

$$DF(\Omega) = \sup \big\{ \eta : \; -(\rho)^\eta \text{ is a bounded plurisubharmonic exhaustion function } \big\} .$$

Diedrich and Fornaess showed that $DF(\Omega)$ is well defined and less or equal to 1 for any smoothly bounded pseudoconvex domain in $\mathbb{C}^n$. Such index has proved to be quite significant in geometric function theory in complex domains.

In this talk I will illustrate several properties of such index and present some recent results obtained in collaboration with Steven Krantz and Bingyaun Liu.

John Ryan (University of Arkansas)
On conformally invariant higher spin Knapp-Stein convolution type operators and associated differential operators

In this work we will look at various types of conformally invariant convolution type operators for higher spin settings. We introduce uncountable infinite such operators and we give a class of conformally invariant differential operators acting as inverses to these operators. We shall use the Cayley transform to carry over these results from euclidean space to the sphere. This is joint work with Chao Ding and Raymond Walter.

Irene Sabadini (Politecnico di Milano)
Differential and integral maps bewteen monogenic and slice monogenic functions

Monogenic functions with values in a Clifford algebra, namely functions in the kernel of the generalized Cauchy-Riemann operator, are widely studied in the literature. They are, in particular, harmonic functions in several variables. Slice monogenic functions (also called slice hyperholomorphic or slice regular functions) have been introduced in more recent times and they have applications, for example, in operator theory. It is thus a natural question to ask if there are relations between the two classes of functions.
Roughly speaking, a function f slice monogenic is of the form $f(u + \underline{\omega} v) =$

$\alpha(u,v) + \underline{\omega}\beta(u,v)$ (where α, β satisfy the Cauchy-Riemann equations and α, β must be even and odd respectively in the second variable v).

The Fueter mapping theorem, which is a classical result in quaternionic and Clifford analysis (which has been investigated by several authors, at various degrees of generality) shows that to any slice monogenic function one may associate a monogenic function of axial type. Specifically, given f slice monogenic, we can represent in integral form the axially monogenic function $\breve{f}(x) = \Delta^{\frac{n-1}{2}} f(x)$ where Δ is the Laplace operator in dimension $n+1$, if $x \in \mathbb{R}^{n+1}$.

We have also solved the inverse problem: given an axially monogenic function $\breve{f}$ we can determine a slice monogenic function f (called Fueter's primitive of $\breve{f}$) such that $\breve{f}(x) = \Delta^{\frac{n-1}{2}} f(x)$. The proof of these results, in the case n even or odd is rather different.

In recent times, we have shown that by weakening a bit the definition of slice monogenic functions it is possible to prove that the Radon transform maps monogenic functions to slice monogenic functions with values in a Clifford algebra and, analogously, the dual Radon transform maps slice monogenic functions to monogenic functions. The proofs of these results are independent of the parity of the dimension.

Simon Salamon (King's College London)
Riemannian twistor theory

I shall present an anthology of results concerning the twistor spaces of S^4 and S^6. This will include work on orthogonal complex structures, and the interpretation of quaternionic polynomials.

David Shoikhet (ORT Braude College & The Technion-Israel Institute of Technology)
Filtration and rigidity of semi-complete vector fields

This talk is based on recent results on generation theory of semigroups of holomorphic mappings with applications to geometric function theory as well as new results which could define some new trends in the development of the subject. We investigate various characterizations, properties and methods of parametric filtration an embedding of the class of semi-complete vector fields (holomorphic generators) and their relations to the special classes of starlike and spirallike functions.

In particular, we show that the class of univalent functions satisfying the Nashiro–Warshawskii condition consists of semi-complete vector fields.

In a parallel manner, we present certain quantitative characteristics of boundary regular null points of semi-complete vector fields and corresponding backward flow invariant domains. In addition, we establish generalized infinitesimal versions of the Burns-Krantz rigidity theorem. Some open questions are discussed.

Vladimir Souček (Univerzita Karlova v Praze)
Construction of a resolution for the k-Dirac operator in even dimensions

The Dirac operator D_k in k variables is a generalization of the Cauchy-Riemann operator in theory of several complex variables to higher dimensions. The equation $D_k(f) = 0$ is an overdetermined system of first order PDEs. In dimension two, the Dolbeault complex is the well-known resolution starting with D_k. The problem how to construct an analogue of the Dolbeault complex in higher dimensions is studied already for several decades.

A complete solution of the problem is known in dimension 4. A series of complexes defined on quaternionic manifolds was constructed by R. Baston in 90's and it includes also the one giving a resolution starting with the Dirac (Fueter) operator in k quaternionic variables. The main tool used in the construction is the Penrose transform developed and described in the book by R. Baston and M. Eastwood.

In general dimensions, several approaches to the problem were developed in the framework of Clifford analysis. The approach to the problem in a general even dimension based on the Penrose transform will be described in the lecture. The resolution is formed by first and second order operators, which are defined on (local) sections of homogeneous vector bundles over a suitable flag manifold G/P (realized as an isotropic Grassmannian). All operators in the resolution are invariant (intertwining) with respect to the action of the group G. The lecture is based on the common work with T. Salač and L. Krump (Charles University, Prague).

Caterina Stoppato (Università di Firenze)
Laurent expansions for slice regular functions and the classification of zeros and singularities

Over the last ten years, function theory over real algebras of dimensions higher than two has undergone significant developments: [Gentili, Struppa 2006] launched the theory of *slice regular* quaternionic functions; [Colombo, Sabadini, Struppa 2009] introduced the theory of *slice monogenic* functions over Clifford algebras; [Ghiloni, Perotti 2011] generalized the notion of slice regularity to real alternative *-algebras and introduced the larger class of *slice functions*. These classes of functions and their applications are intensively studied by the same authors and collaborators.

Some recent results concern the algebraic structure and the zero sets of slice functions. These, in turn, allow the introduction a new type of series expansion near each singularity of a slice regular function, leading to a complete classification of singularities. Peculiar phenomena arise, which are not present in the complex case and grow articulate as the hypotheses on the algebra are relaxed. These recent results have been obtained jointly with Riccardo Ghiloni and Alessandro Perotti (Università di Trento).

Alberto Verjovsky (UNAM, Mexico)
Modular groups over the quaternions and their corresponding four dimensional hyperbolic orbifolds and manifolds

Using the rings of Lipschitz and Hurwitz integers in the division algebra of the quaternions we define several Kleinian discrete subgroups of isometries of hyperbolic 4-space. These groups are quaternionic versions of the classical modular group and Picard group. The quotients of hyperbolic 4-space by the action of these groups give examples of arithmetic orbifolds which are finitely covered by very interesting arithmetic hyperbolic 4-manifolds of finite volume.

Alain Yger (Université de Bordeaux)
Currents in algebraic or arithmetic geometry; how to exploit "ghost" antiholomorphic coordinates

I will present an overview on the role of currents (*i.e* differential forms with distribution coefficients) towards effectivity questions in analytic, algebraic or even arithmetic geometry, focusing both on positive and negative points. I will for example explain how such tools (or analytic nature) unfortunately involve averaging procedures (see for example Bochner-Martinelli versus Cauchy kernel approaches), which may become a stumbling block with respect to questions of arithmetic nature. I will nevertheless insist on the key role played by the duplication of holomorphic coordinates into anti-holomorphic "ghost" ones. Recent developments in non-archimedean analytic geometry (tropicalization, realization of (p, q)-currents and d', d'' operators in the real setting) will be presented as a corroboration of such facts.