FINAL REPORT ON THE PROJECT QUASRFUN

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Outgoing host instititution: Institut de Mathématiques de Jussieu - Paris Rive Gauche (IMJ-PRG)
Period: 1 September 2015 - 30 September 2016

The project *Quaternionic slice regular functions, applications and related topics - QuaSRFun* is placed in the area of complex and hypercomplex analysis and geometry. More precisely, on the one hand it concerns the theory of slice regular functions of one quaternionic variable and its possible applications to different areas of mathematics. On the other hand it deals with the study of holmorphic function spaces in two complex variables, as the Dirichlet space on the complex bidisc.

Report (1 September 2015 - 30 September 2016)

During these 13 months I worked within the "Complex Analysis and Geometry" group at the IMJ-PRG, continuing as well my italian collaborations. I participated in several scientific events (both in France and in Italy), and I gave several talks (both in France and in Italy). The main research themes I started and/or continued to develop are:

- Quaternionic manifolds: study of quaternionic *slice regular* manifolds also aimed to the comprehension of the possible connections between slice regularity and hyperKähler and quaternion Kähler geometry. The first basic results in this setting are contained in the paper [2], a joint work with G. Gentili (University of Florence) and A. Gori (University of Milan).

- Quaternionic toric manifolds: 4-plectic viewpoint, study of the convexity of these objects, search for a construction à la Delzant starting from appropriate polytopes. The results of this research are contained in the preprint [4], a joint work with G. Gentili and A. Gori.

- Study of the Dirichlet space on the complex bidisc (and polydiscs), characterization of Carleson measures for this space, relation with potential theory in the discrete setting. A first result of this research is the paper [3], a joint work with N. Arcozzi (University of Bologna), P. Mozolyako (University of St. Petersburg), K.M. Perfekt (University of Trondheim), S. Richter (University of Tennessee).

- In collaboration with Elisha Falbel and Marco Maculan of the IMJ-PRG, we are studying the space of configurations of flags in the orbit of real forms of the complex Lie group $SL(4, \mathbb{C})$. Our initial motivation was the study of quaternionic flags and their space of configurations, aimed to find a quaternionic analogue of the volume map for tetrahedra of flags.

- In collaboration with Antonin Guilloux of the IMJ-PRG, we are studying the space of deformations of representations in $SU(2, 1)$ in the quaternionic group $Sp(2, 1)$. Our starting point is the study of a particular representation of the fundamental group of the figure eight knot complement.

Conferences, Workshops and Schools attended

- *XX Congresso dell'Unione Matematica Italiana*, Siena, 7th-12th September 2015.
  INVITED TALK: "Metriche invarianti associate allo spazio di Hardy quaternionico".

- *Hypercomplex Analysis and Geometry*, Ferrara, 1st December 2015.

- *Workshop Varietà reali e complesse: geometria, topologia e analisi armonica*, Scuola Normale Superiore, Pisa, 21st - 23rd January 2016.

- *Special Hermitian metrics on non-Kähler manifolds*, Firenze, 20th-22nd April 2016.

- *Spring 2016 School*, Firenze, 6th - 10th June 2016.
  Minicourses: "Complex structures, spinors and projective geometry" by Simon Salamon and "Slice regular functions on real alternative $*$-algebras" by Riccardo Ghiloni and Alessandro Perotti.

- *13th Advanced Course in Operator Theory and Complex Analysis*, Lyon, 13th - 15th June 2016.
  CONTRIBUTED TALK: "About Carleson measures for the Dirichlet space on the bidisc".

- *Differential Geometry in the Large*, Firenze, 11th-15th July 2016.

- *Complex function theory, its generalizations and applications*, INdAM, Roma, 12th-16th September 2016.

Seminars

- 29th January 2016, *L'espace de Hardy quaternionique et la géométrie de la boule unité*, Séminaire Gaston Darboux, Institut Montpelliérain Alexander Grothendieck, Université de Montpellier 2.

- 17th May 2016, *L'espace de Hardy quaternionique et la géométrie de la boule unité*, Séminaire d'analyse complexe et géométrie, Institut de Mathématiques de Jussieu - Paris Rive Gauche, Université de Paris 6.

Organized events

Member of the organizing committee of the Workshop *Complex function theory, its generalizations and applications*, at INdAM, Roma, 12th-16th September 2016.

Hosted visitors

28th March - 1st April 2016: Graziano Gentili (University of Florence) and Anna Gori (University of Milan).

Referee activity

I have been reviewer for The Journal of Geometric Analysis, Mathematische Nachrichten and Advances in Applied Clifford Algebras.

Publications and preprints

1. G. SARFATTI, *The quaternionic Hardy space and the geometry of the unit ball*, Bruno Pini Mathematical Analysis Seminar (2015) doi: 10.6092/issn.2240-2829/5893.

2. G. GENTILI, A. GORI, G. SARFATTI, *A direct approach to quaternionic regular manifolds*, on-line first in Mathematische Nachrtichten, doi: 10.1002/mana.201500489.

3. N. ARCOZZI, K.M. PERFEKT, P. MOZOLYAKO, S. RICHTER, G. SARFATTI, *Some Hilbert Spaces related with the Dirichlet space*, Concrete Operators **3** (2016), doi: 10.1515/conop-2016-0011.

4. G. GENTILI, A. GORI, G. SARFATTI, *Quaternionic toric manifolds*, submitted for publication, preprint avaliable at https://dl.dropboxusercontent.com/u/15440756/toricJSG.pdf