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Subject: Final report, 2-year postdoc INdAM-COFUND-2012

1. Introduction

I studied mathematics at University of Oriente in Santiago de Cuba. I graduated with the highest grade in the spring of 2002 and began to work as a teaching assistant at University of Holguín. Approximately one year later, I obtained a Ph.D. fellowship from Ghent University in Belgium.

During the Ph.D. period (fours years, 2004-2008), I worked under the supervision of Prof. Frank Sommen who is a leading researcher in the field of Clifford analysis. On June 6 of 2008, I received my academic degree with the thesis entitled *"Cauchy–Kowalevski extensions, Fueter's theorems and boundary values of special systems in Clifford analysis"*.

From 2008 to 2014, I have been a postdoctoral researcher at the University of Aveiro, Portugal (3 years, 2008-2011) and also at Ghent University, Belgium (3 years, 2011-2014). This was possible thanks to postdoctoral grants awarded by the *Foundation for Science and Technology* in Portugal and by the *Special Research Fund* from Ghent University respectively. By now I have published 39 articles in international journals, 2 contributions in edited volumes and 4 contributions in proceedings. I have also presented my scientific results in many scientific congresses and conferences.

From 1 November 2014 until 31 October 2016, I had the honour of being a Marie Curie Fellow of the *Istituto Nazionale di Alta Matematica, INdAM*. The research was carried out at the Department of Mathematics of the Politecnico di Milano in collaboration with Prof. Irene Sabadini and Prof. Fabrizio Colombo. The title of the research project was: *"Exact solutions of systems of Dirac equations in real and complex Clifford analysis"*. In what follows, I describe my activities during the two years of the INdAM-COFUND fellowship.

2. Research

The area of research is Clifford analysis, which can be described as a higher dimensional function theory for functions defined on Euclidean spaces and taking values in a Clifford algebra, the latter being a generalization of the complex numbers and Hamilton's quaternions.

The use of Clifford algebras allows for the consideration of various Clifford algebra-valued differential operators and systems of partial differential equations, which are important in mathematics and mathematical physics. The most fundamental one is the Dirac equation for left monogenic functions, providing a higher dimensional generalization of the theory of holomorphic functions in the complex plane. This framework also allows to study other interesting systems of partial differential equations including the Riesz system, the Hodge system, Maxwell's equations, the equations for two-sided (left and right) monogenic functions and refinements of these systems, such as Hermitian Dirac systems, which give rise to a function theory (called Hermitian Clifford analysis) related to and even encompassing several complex variables theory.

In this research we focussed on special functions and partial differential systems that arise naturally within the framework of Clifford analysis.

During the first year of the INdAM-COFUND fellowship, we have worked on the two-sided monogenic system and the so-called Hermitian submonogenic system, which we have already introduced in the setting of Hermitian Clifford analysis.

For the first system we study axially symmetric solutions and present three methods to obtain special solutions. Each of these methods is effective enough to generate all the polynomial solutions. In the research related with the Hermitian submonogenic system, we investigate plane wave solutions, leading to the construction of a Cauchy kernel. We also establish a Stokes type formula that, when applied to the Cauchy kernel provides an integral representation formula for Hermitian submonogenic functions.

In the second year of the fellowship we continue with the study on the Hermitian submonogenic system. We also worked on topics such as the Segal-Bargmann transform and Fueter's theorem. The latter is a fundamental result in Clifford analysis which generates special left monogenic functions.

More precisely, we characterize all the polynomial solutions of the Hermitian submonogenic system. Next, we consider the Segal-Bargmann transform in the framework of Clifford algebra-valued functions and we study monogenic Bargmann modules. We also prove that the Segal-Bargmann kernel corresponds to the kernel of the Fourier-Borel transform for monogenic functionals. Finally, we obtain an important geneneralization of Fueter's theorem in the setting of monogenic functions in biaxially symmetric domains.

3. Papers and scientific activities

During the scholarship we have written five articles.

1. F. Colombo, D. Peña Peña, F. Sommen, A Cauchy kernel for the Hermitian submonogenic system, *Mathematische Nachrichten* (2016), DOI: 10.1002/mana.201500451 (http://onlinelibrary.wiley.com/doi/10.1002/mana.201500451/full).

2. D. Peña Peña, I. Sabadini, F. Sommen, Polynomial solutions of the Hermitian submonogenic system, *Journal of Mathematical Analysis and Applications* 446 (2017), no. 1, 730-745 (http://www.sciencedirect.com/science/article/pii/S0022247X1630511X).

3. D. Peña Peña, I. Sabadini and F. Sommen, On two-sided monogenic functions of axial type, to appear in *Moscow Mathematical Journal*, arXiv:1609.07794 (https://arxiv.org/pdf/1609.07794v1.pdf).

4. D. Peña Peña, I. Sabadini and F. Sommen, Segal-Bargmann-Fock modules of monogenic functions, submitted to *Journal of Mathematical Physics*, arXiv:1608.06790 (https://arxiv.org/pdf/1608.06790v1.pdf).

5. D. Peña Peña, I. Sabadini and F. Sommen, Fueter's theorem for monogenic functions in biaxial symmetric domains, submitted to *Arkiv för Matematik*, arXiv:1611.01324 (https://arxiv.org/pdf/1611.01324v1.pdf).

Attending international meetings is of vital importance for every researcher since they are sources of new ideas and inspiration. I have taken part in the following meetings.

1. 17th European Workshop on Applications and Generalizations of Complex Analysis, University of Aveiro, Portugal, 20-21 March 2015. Presentation of: *Fueter's theorem versus axial monogenics*.

2. Joint International Meeting of the American, European and Portuguese Mathematical Societies, University of Porto, Portugal, 10-13 June 2015. Presentation of: *Biaxial monogenic functions from Funk-Hecke's formula combined with Fueter's theorem*.

3. Workshop Non-commutative Algebras and Applications, 14-16 June 2015, Goslar, Germany. Presentation of: *The Hermitian submonogenic system*.

4. 36th Winter School Geometry and Physics, Czech Republic, Srni, January 16-23, 2016. Presentation of: *An integral representation formula for the Hermitian submonogenic functions*.

5. International Conference *Past and Future Directions in Hypercomplex and Harmonic Analysis*, Universidade de Aveiro, March 29-April 2, 2016. Presentation of: *Some special functions in real and complex Clifford analysis*.

Prof. Irene Sabadini and myself have organized the following scientific activities at the Politecnico di Milano with the support of the RCC1 funding:

- *Clifford analysis techniques in image processing*, series of lectures imparted by Prof. Uwe Kähler and Prof. Paula Cerejeiras from University of Aveiro (Portugal), 23-27 November 2015.

- *Spherical monogenics in dimension 3 and discrete orthogonal polynomials*, Uni- and *multivariate discrete orthogonal polynomials using Dirac operators*, two lectures imparted by Prof. Hendrik De Bie from Ghent University (Belgium), 9-13 May 2016.

Thanks to the RCC1 funding, it was also possible to finance several research stays of Prof. Frank Sommen at the Politecnico di Milano. These stays were essential for the completion of the above articles.

4. Conclusion

I would like to mention that the INdAM-COFUND fellowship has given me the opportunity to improve and expand my skills in my area of expertise and to gain knowledge in other research fields. It has also been a very important experience in my professional and personal life. I am therefore very grateful to INdAM for the financial support and the Department of Mathematics of the Politecnico di Milano for the excellent conditions of work and nice environment to do research.

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